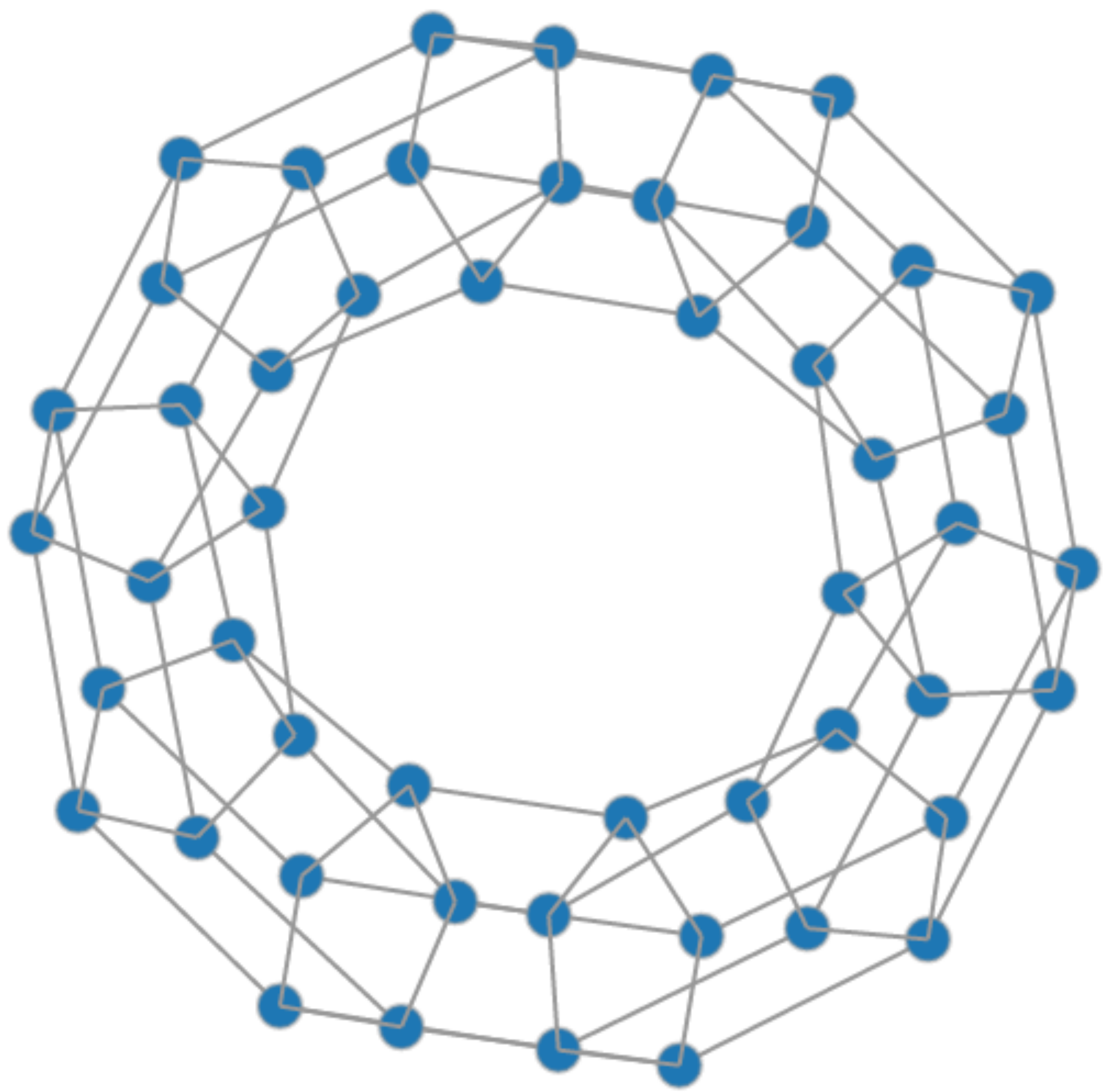
Graphs

CSC444

Node-link diagrams



http://christophermanning.org/gists/1703449/#/%5B10%5D50/1/0

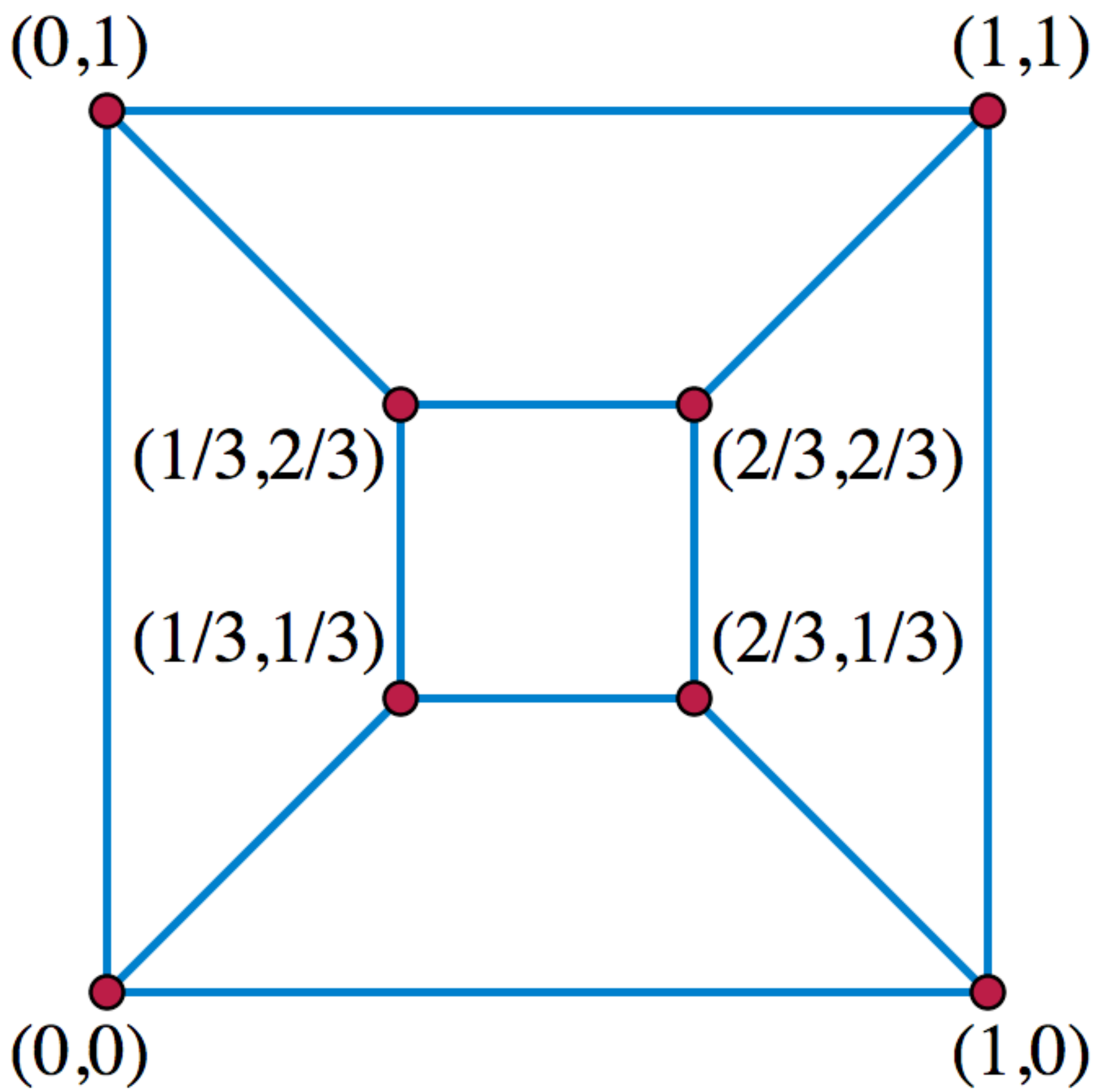
Starting simple: planar 3-vertex connected graphs (what?)

Tutte Embedding

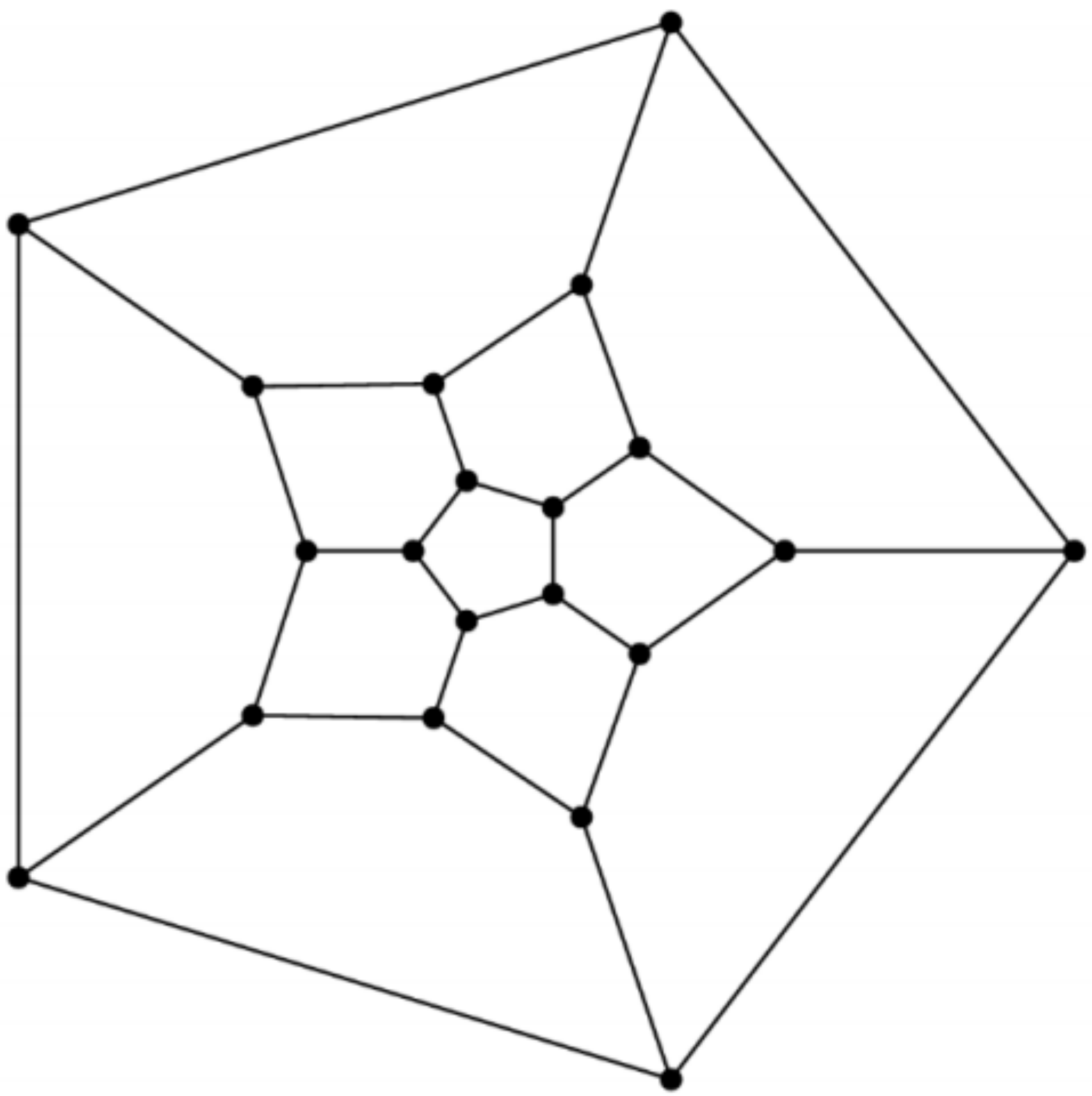
- Each node should be the average of its neighbors
  - Aside from the boundary, which is user-specified
- This gives a linear system of equations

- Theorem: **if graph is planar, embedding is crossing-free**

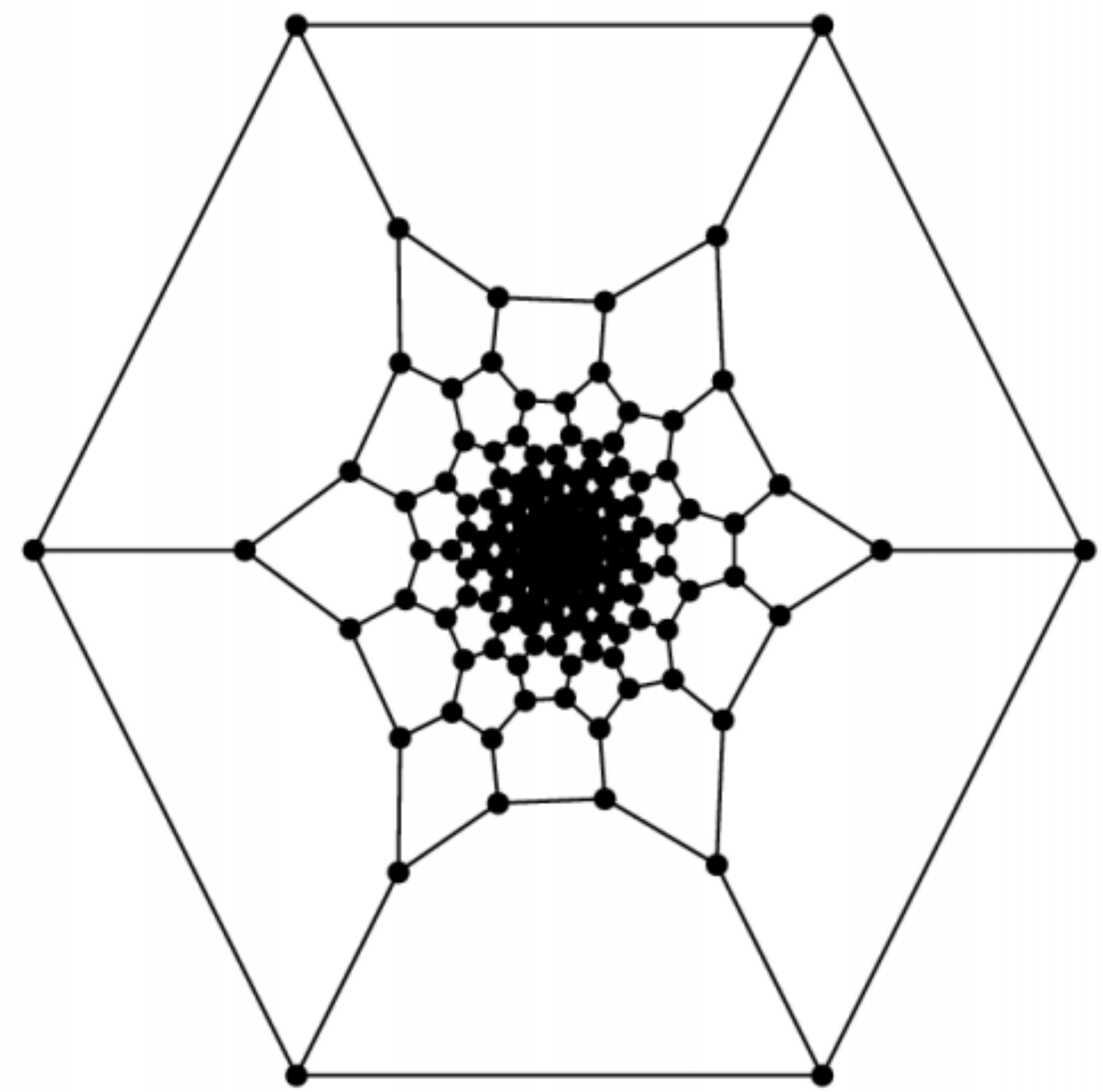
Tutte Embedding



Downsides



Dodecahedron



$Le(C60)$

http://www.cs.arizona.edu/~kpavlou/Tutte_Embedding.pdf

Force-directed Layouts

- Intuition: define “forces” on “physical objects”, initialize positions randomly, let the system settle

http://bl.ocks.org/mbostock/4062045

- Need to define what forces are, and what physical objects are

Force-directed Layouts

- We want edges to be neither too small or too large
  - Aesthetic principle: graph neighbors should be close
  - Physical analogy: **Springs compress or expand to achieve ideal length**
- We don’t want vertices to bunch up together
  - Aesthetic principle: position in screen should be unambiguous indicator of vertex identity
  - Physical analogy: **Electric charges with the same sign don’t bunch up**

Force-directed Layouts

- Force per edge: $f_E(d) = C_E \times (d - L)$

Force-directed Layouts

- Force per vertex pair: $f_V(d) = C_V \times \frac{m_1 m_2}{d^2}$

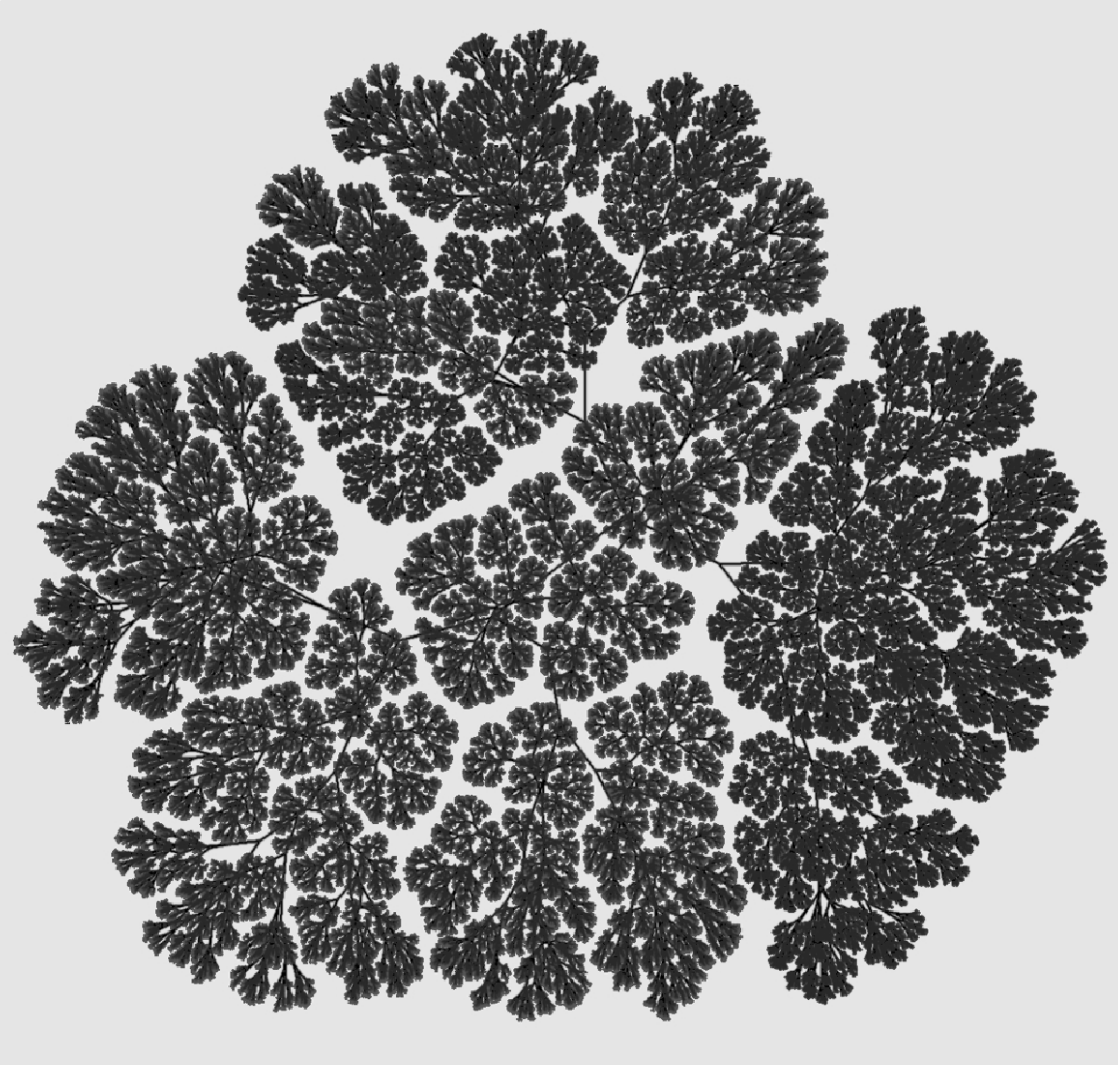
Force-directed Layouts

- Algorithm:
- For each vertex, determine all forces that apply to it,
  - Edges $f_E(d) = C_E \times (d - L)$
  - vertices $f_V(d) = C_V \times \frac{m_1 m_2}{d^2}$
- compute direction of movement, move small amount in those directions
  - iterate until convergence

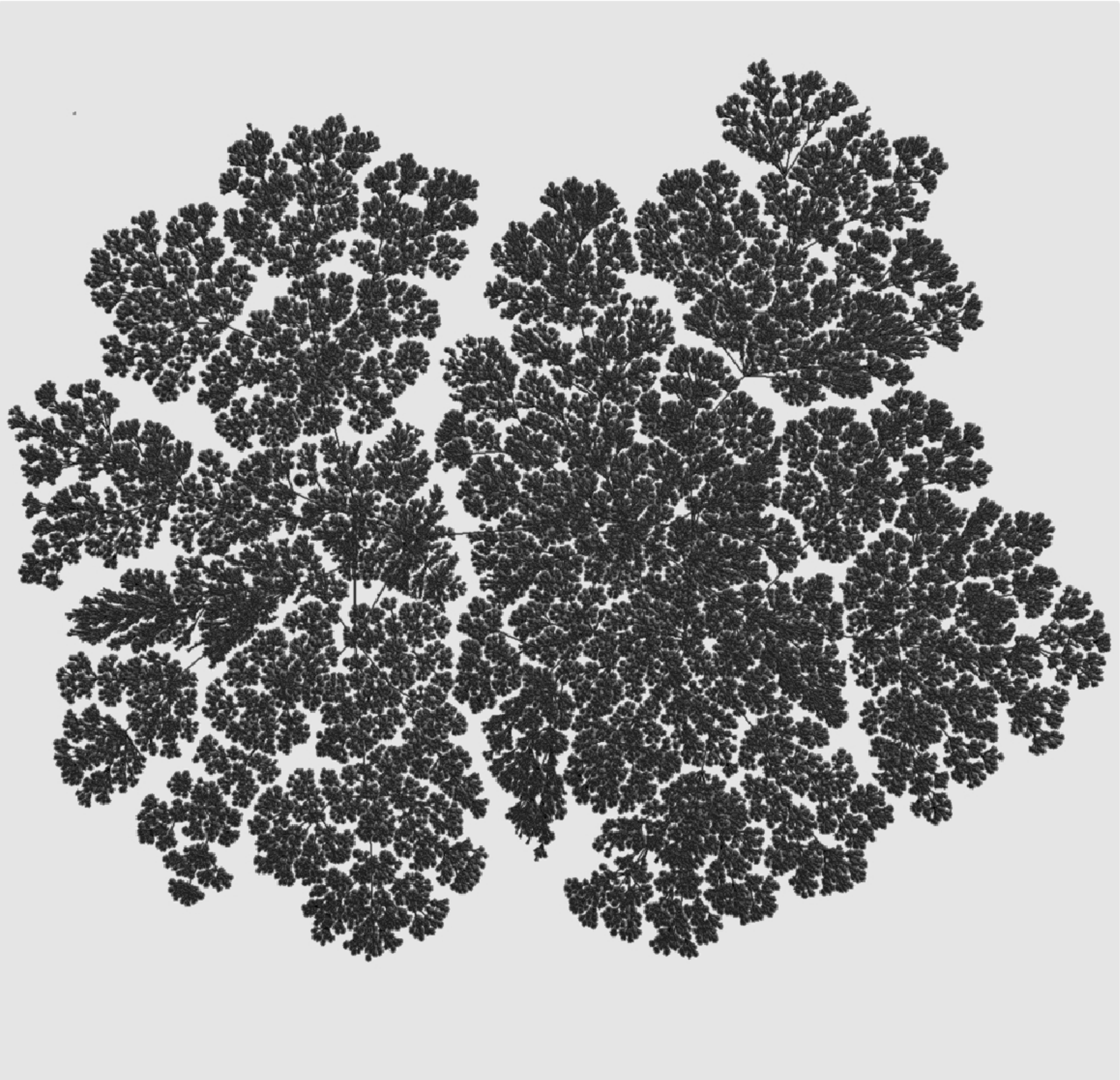
Downsides

- Requires $O(|V|^2)$ work per step
  - Faster algorithms exist: Barnes-Hut, multipole methods, etc.
- For large graphs, result is not very informative

Downsides



Downsides

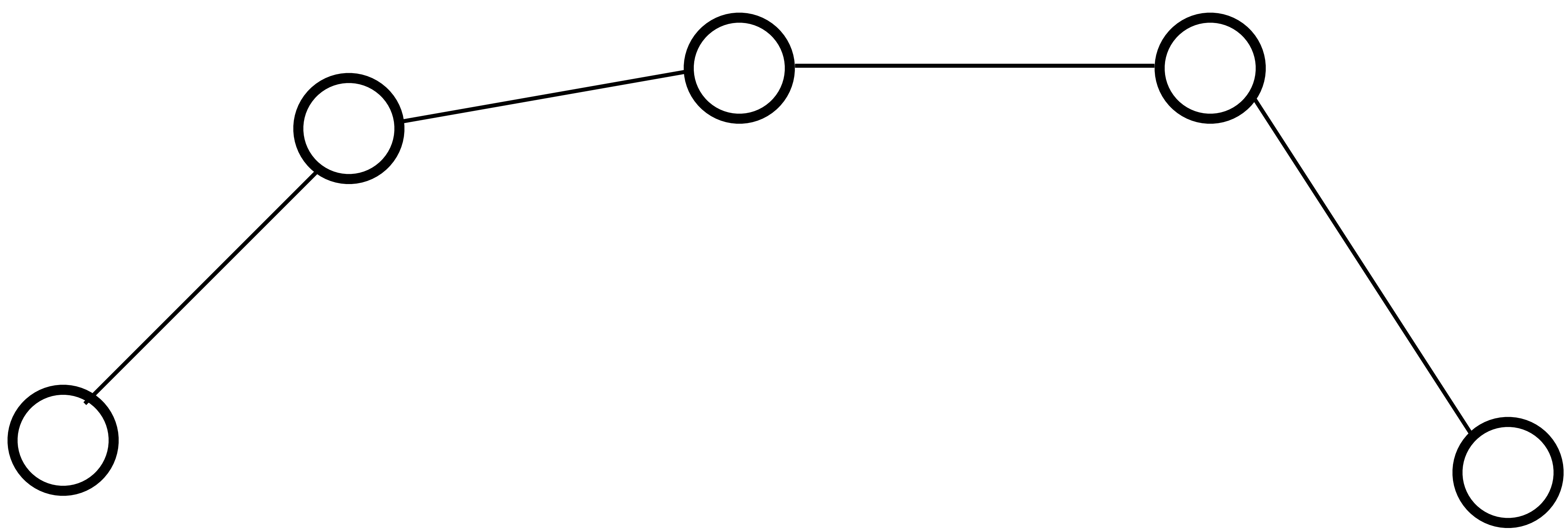


Metric Embeddings

- Use **global properties** of the graph instead of only local interactions
- Specifically, graph **distances**

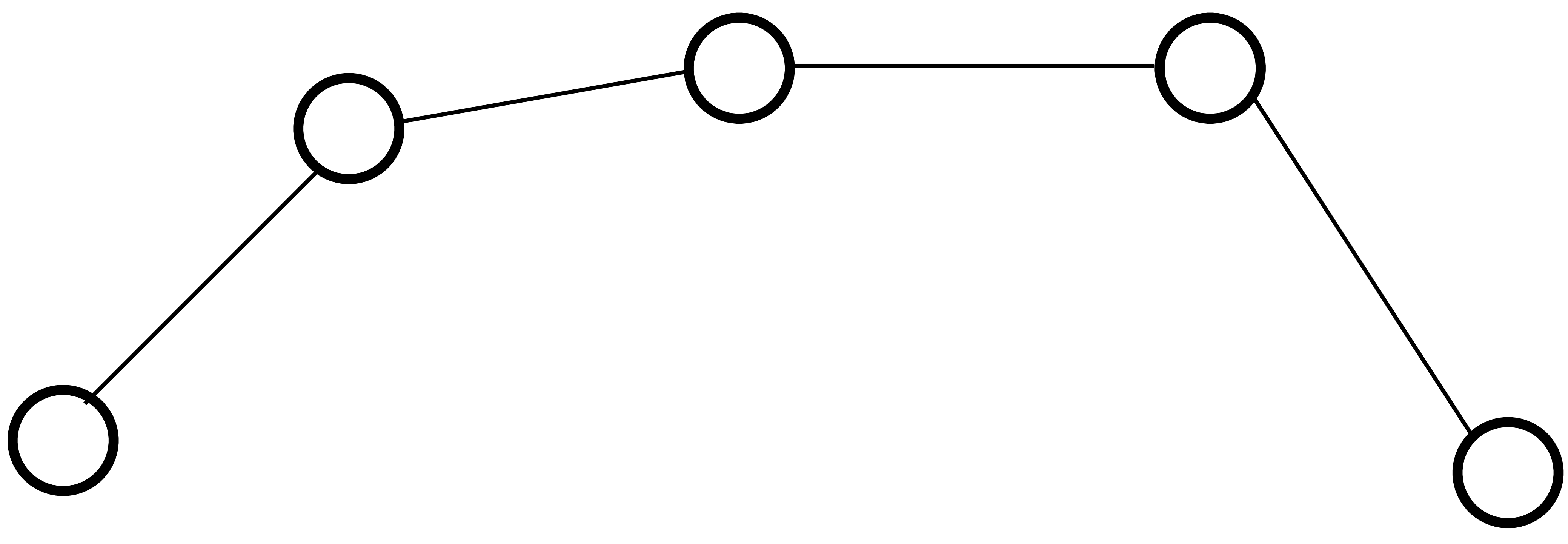
Metric Embeddings

- Graph distances can be used to define “forces”
    - Encode directly that **far away vertex pairs should be placed far from one another**



Metric Embeddings

$$E(X) = \sum_{i,j} (d(i,j) - |X_i - X_j|)^2$$

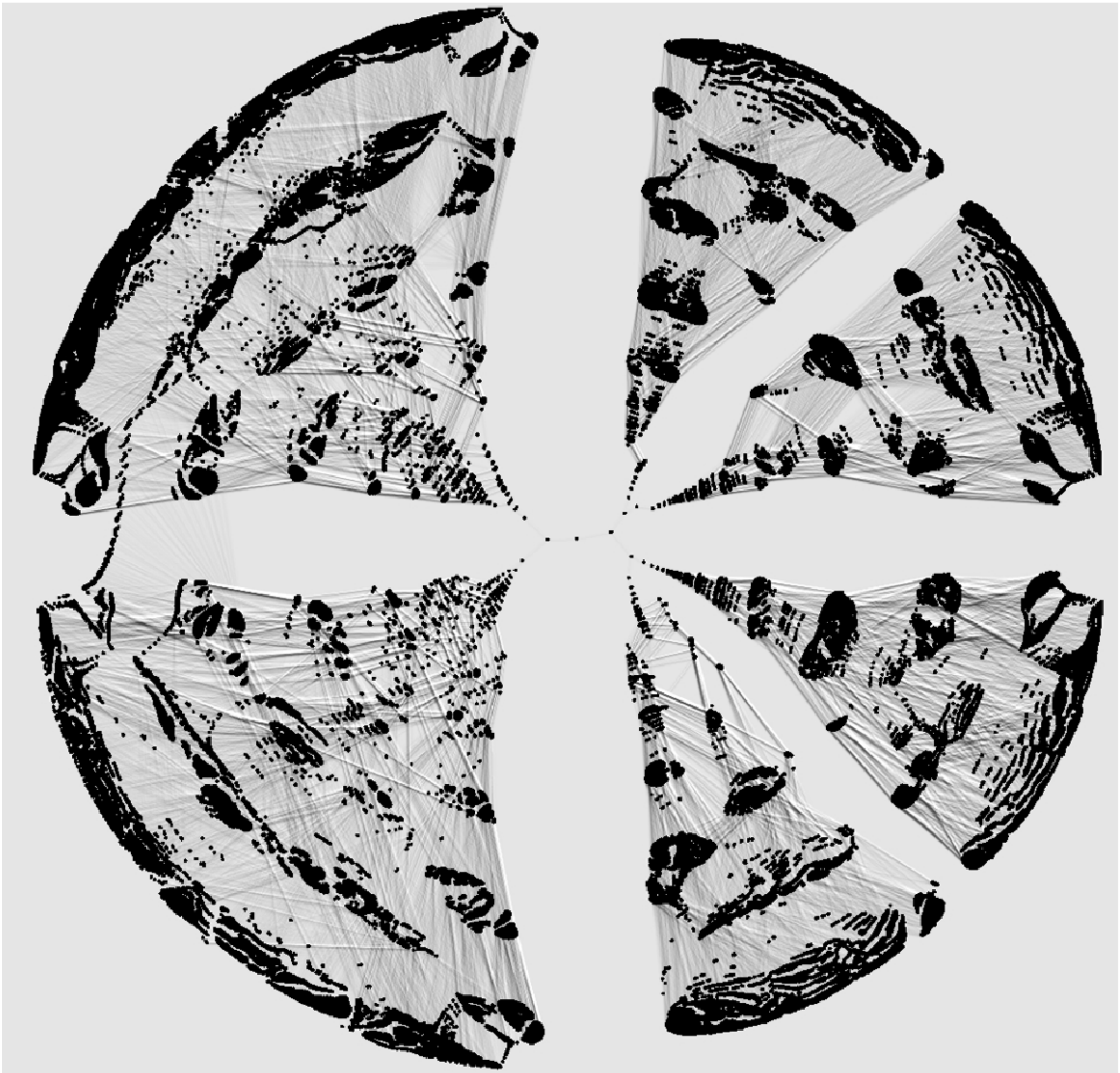


Metric Embeddings

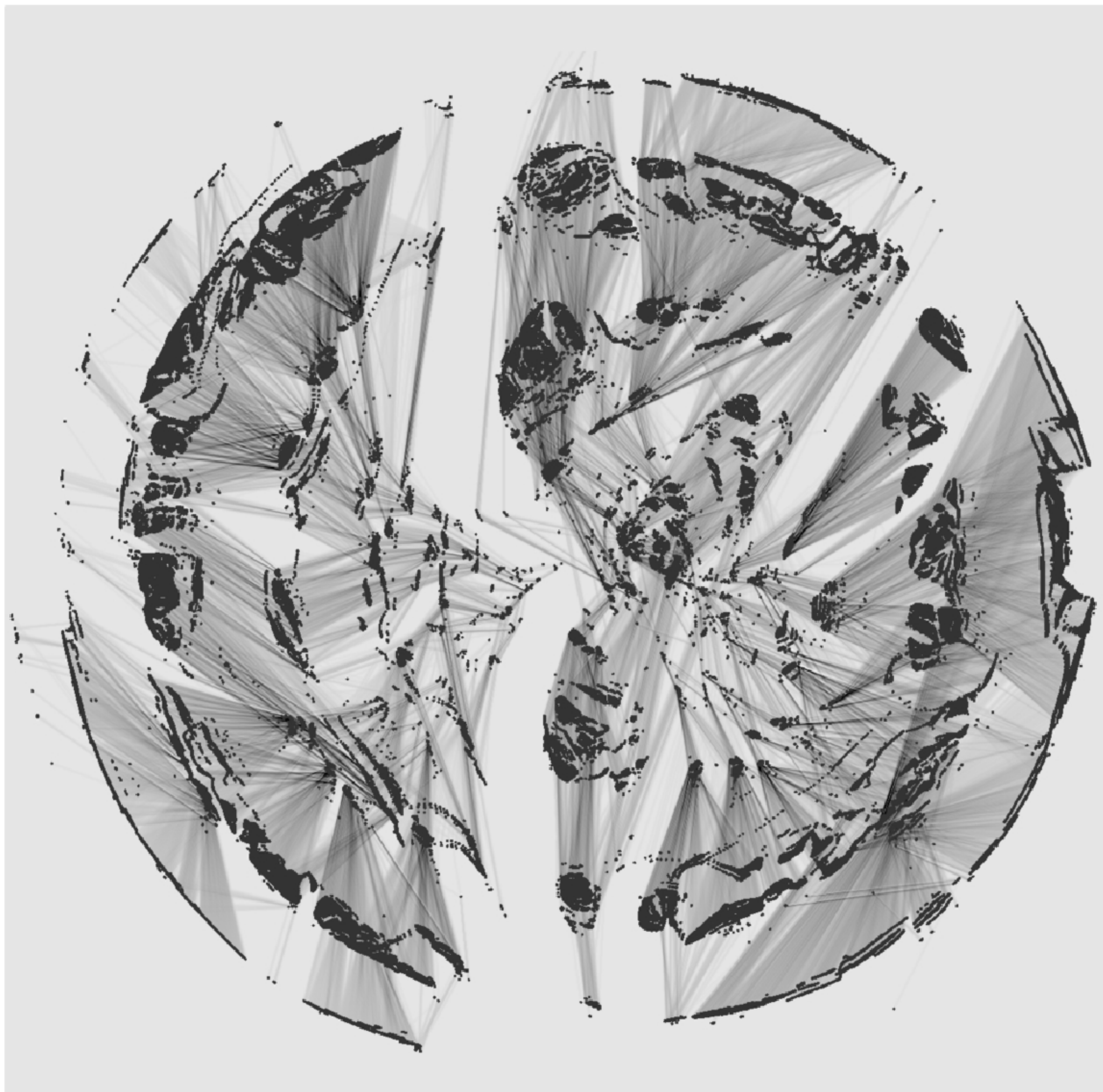
$$E(X) = \sum_{i,j} (d(i,j) - |X_i - X_j|)^2$$

- Our old friend, dimensionality reduction!

Metric Embeddings

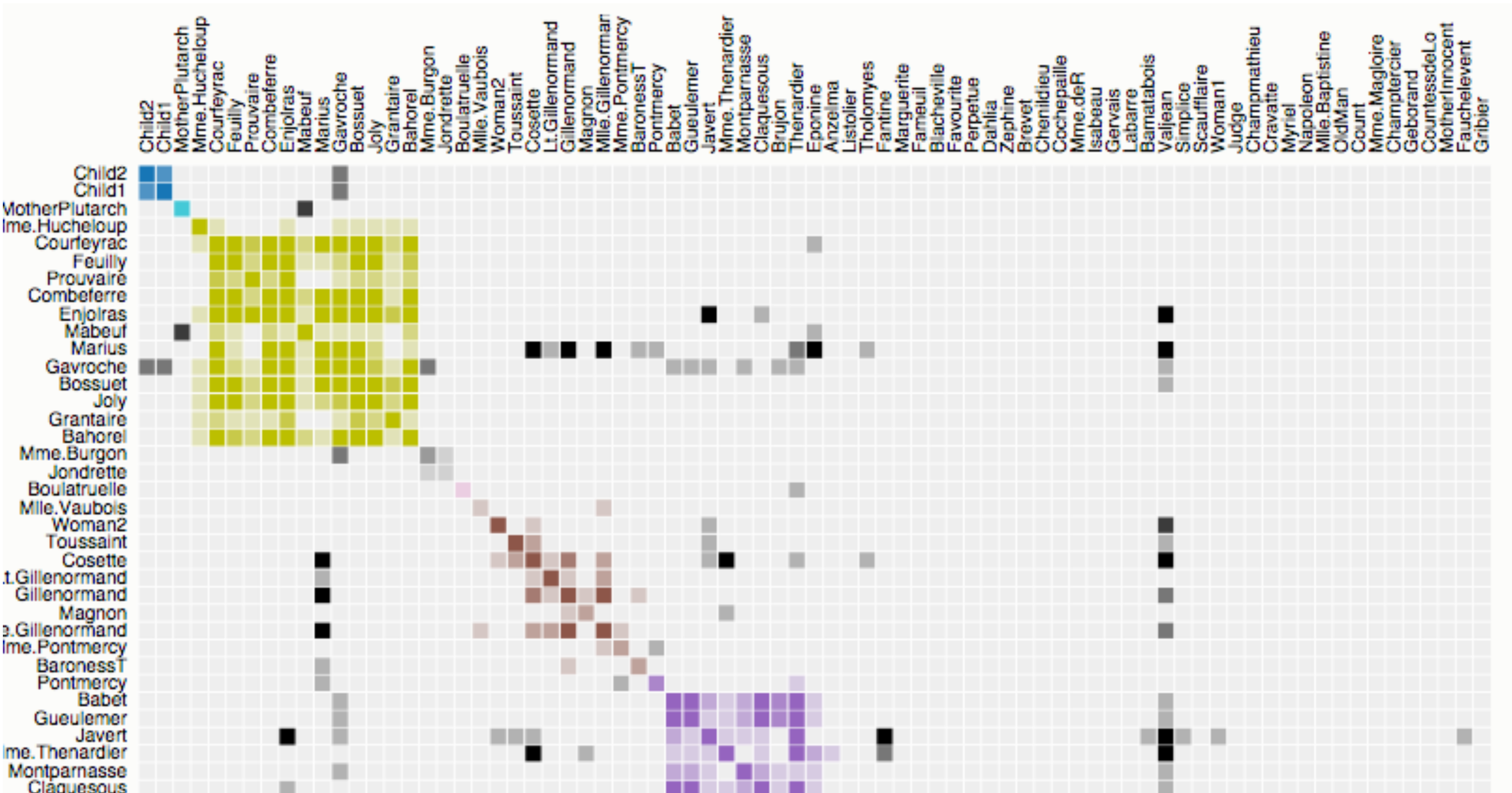


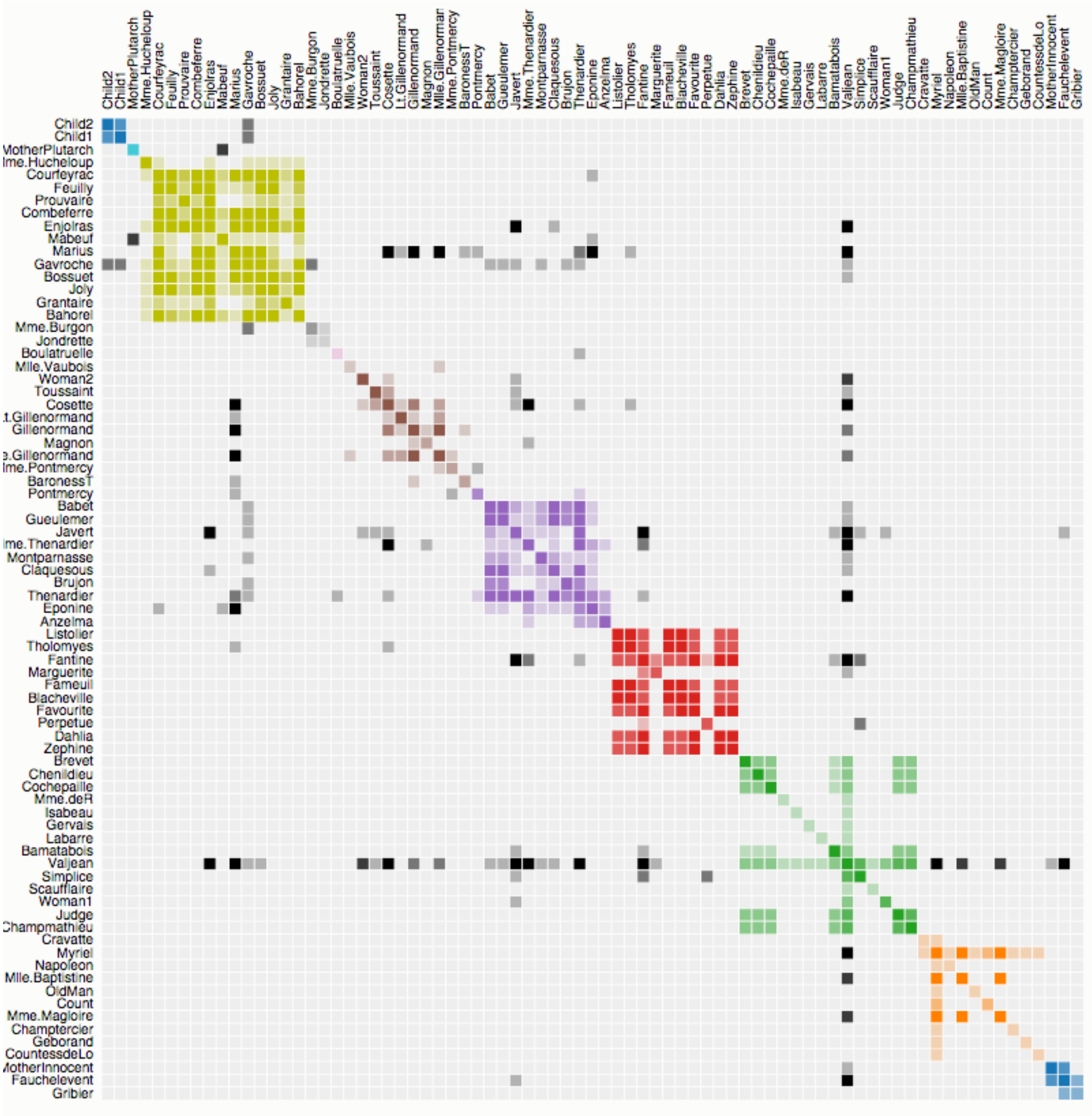
Metric Embeddings



Matrix Diagrams

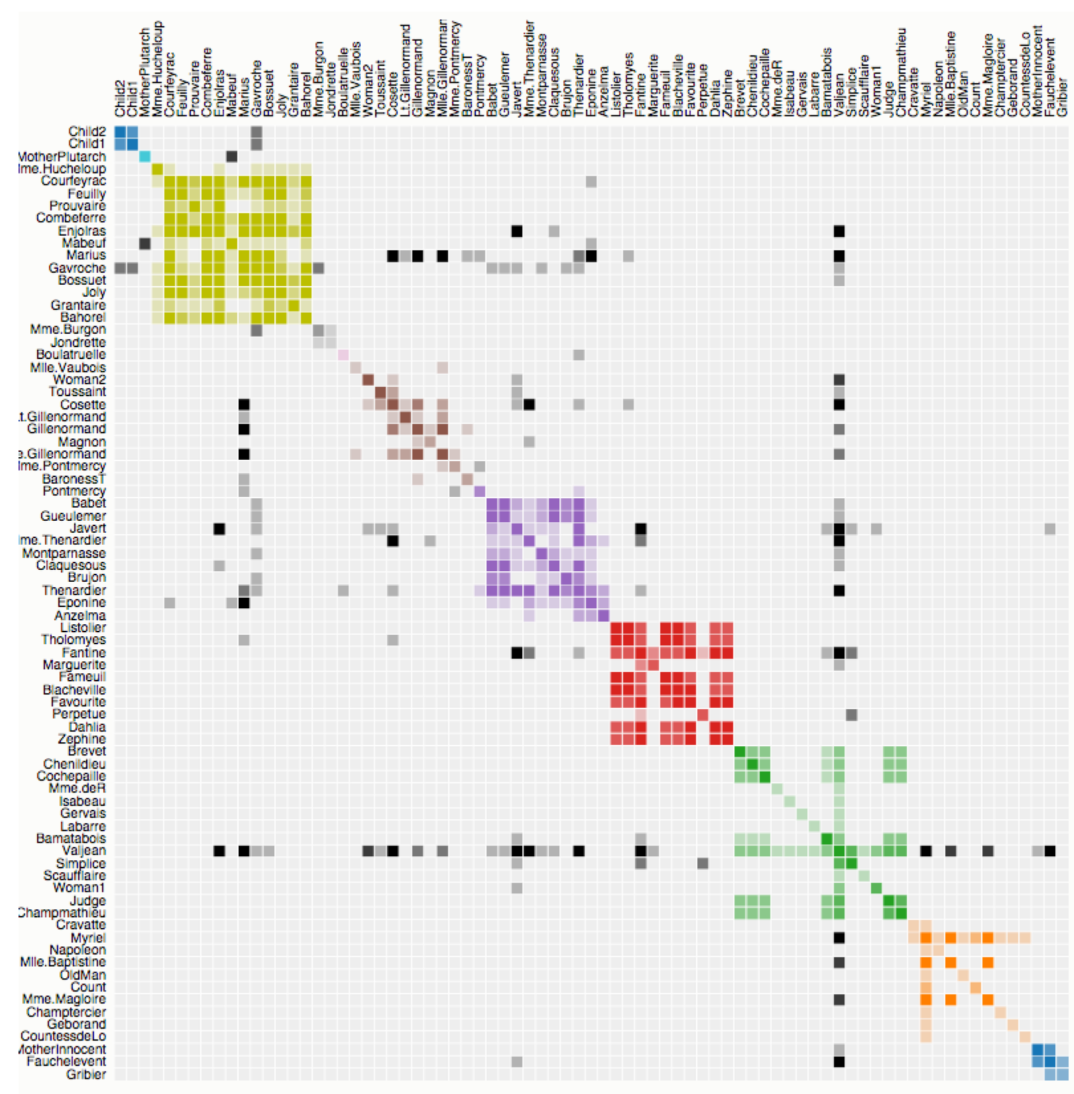
http://bost.ocks.org/mike/miserables/




Child2
Child1
MotherPlutarch
Mme.Hucheloup
Courfeyrac
Feuilly
Prouvaire
Combeferre
Enjolras
Mabeuf
Marius
Gavroche
Bossuet
Joly
Grantaire
Bahorel
Mme.Burgon
Jondrette
Boulatruelle
Mlle.Vaubois
Woman2
Toussaint
Cosette
Lt.Gillenormand
Gillenormand
Magnon
Mlle.Gillenormar
Mme.Pontmercy
BaronessT
Pontmercy
Babet
Gueulemer
Javert
Mme.Thenardier
Montparnasse
Claquesous
Brujon
Thenardier
Eponine
Anzelma
Listolier
Tholomyes
Fantine
Marguerite
Fameuil
Blacheville
Favourite
Perpetue
Dahlia
Zephine
Brevet
Chenildieu
Cochepaille
Mme.deR
Isabeau
Gervais
Labarre
Bamatabois
Valjean
Simplice
Scaufflaire
Woman1
Judge
Champmathieu
Cravatte
Myriel
Napoleon
Mlle.Baptistine
OldMan
Count
Mme.Magloire
Champtercier
Geborand
CountessdeLo
MotherInnocent
Fauchelevent
Gribier
Child2
Child1
MotherPlutarch
lme.Hucheloup
Courfeyrac
Feuilly
Prouvaire
Combeferre
Enjolras
Mabeuf
Marius
Gavroche
Bossuet
Joly
Grantaire
Bahorel
Mme.Burgon
Jondrette
Boulatruelle
Mlle.Vaubois
Woman2
Toussaint
Cosette
.t.Gillenormand
Gillenormand
Magnon
e.Gillenormand
lme.Pontmercy
BaronessT
Pontmercy
Babet
Gueulemer
Javert
lme.Thenardier
Montparnasse
Claquesous
Brujon
Thenardier
Eponine
Anzelma
Listolier
Tholomyes
Fantine
Marguerite
Fameuil
Blacheville
Favourite
Perpetue
Dahlia
Zephine
Brevet
Chenildieu
Cochepaille
Mme.deR
Isabeau
Gervais
Labarre
Bamatabois
Valjean
Simplice
Scaufflaire
Woman1
Judge
Champmathieu
Cravatte
Myriel
Napoleon
Mlle.Baptistine
OldMan
Count
Mme.Magloire
Champtercier
Geborand
CountessdeLo
MotherInnocent
Fauchelevent
Gribier

Upsides

- Easy to define for directed and undirected graphs
- Easy to compute
- Easy to incorporate edge attributes



Downsides

- The order in which rows are chosen makes a big impact in the visualization