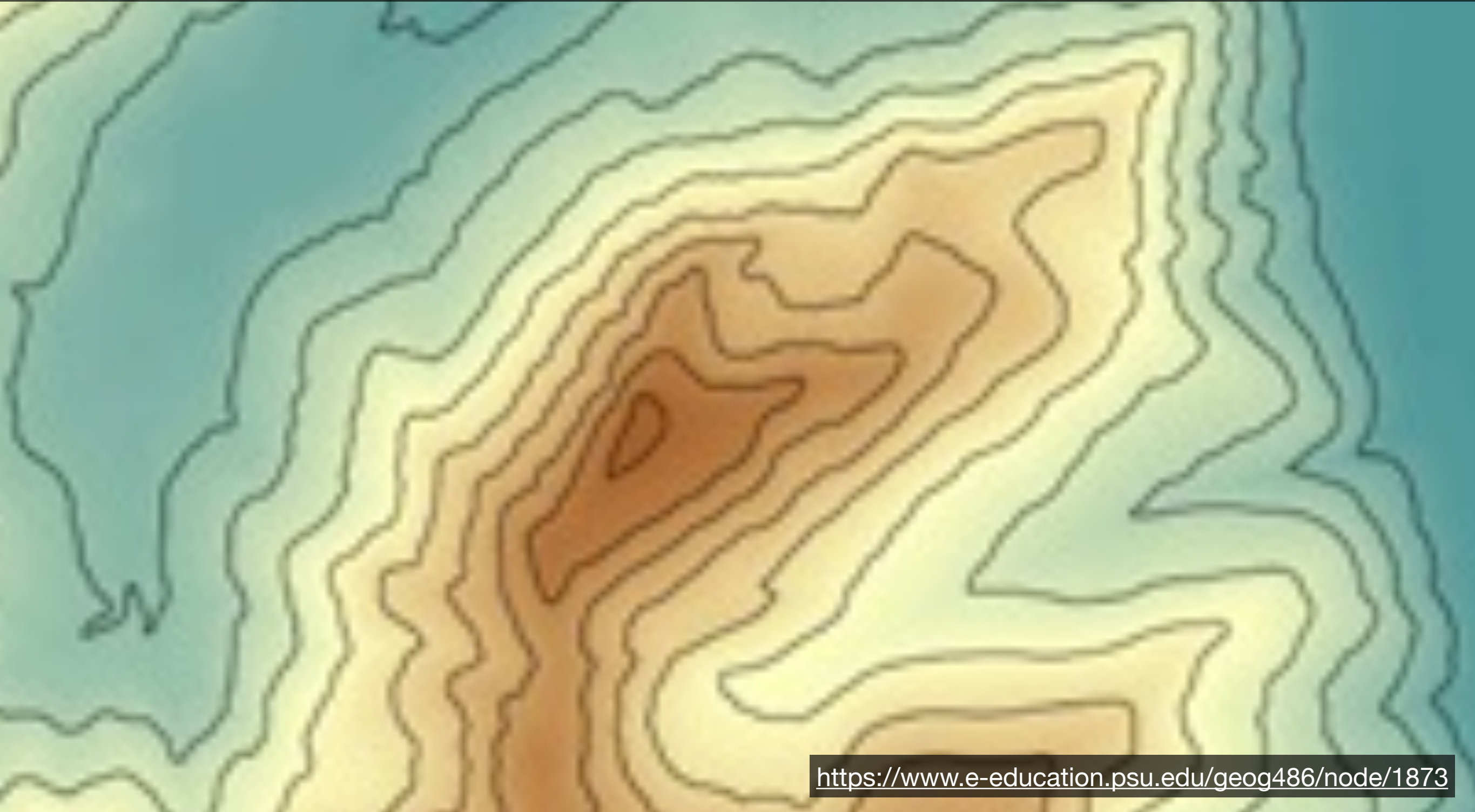


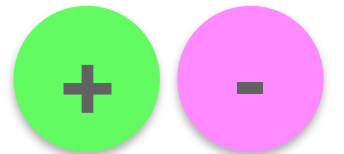
# Spatial Data: 3D Scalar Fields

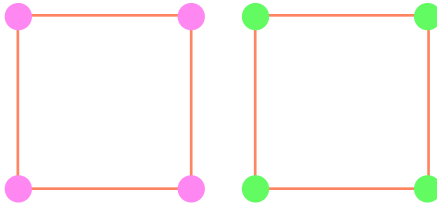
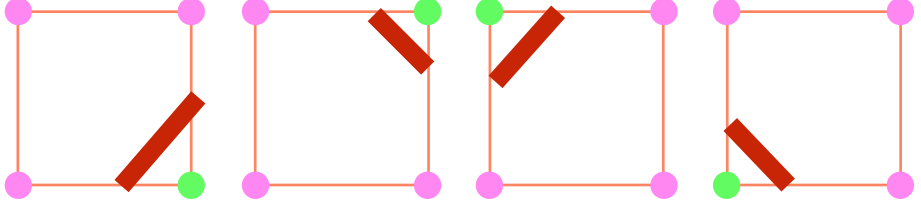
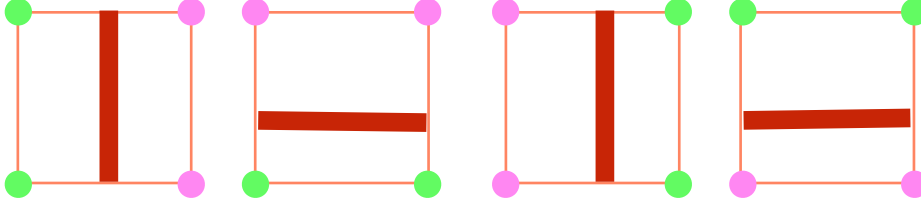
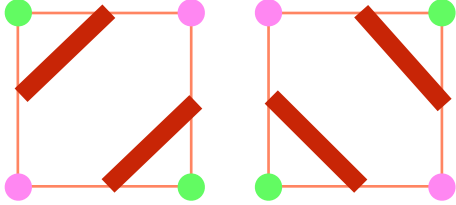
CSC444

# Recap: 2D contouring

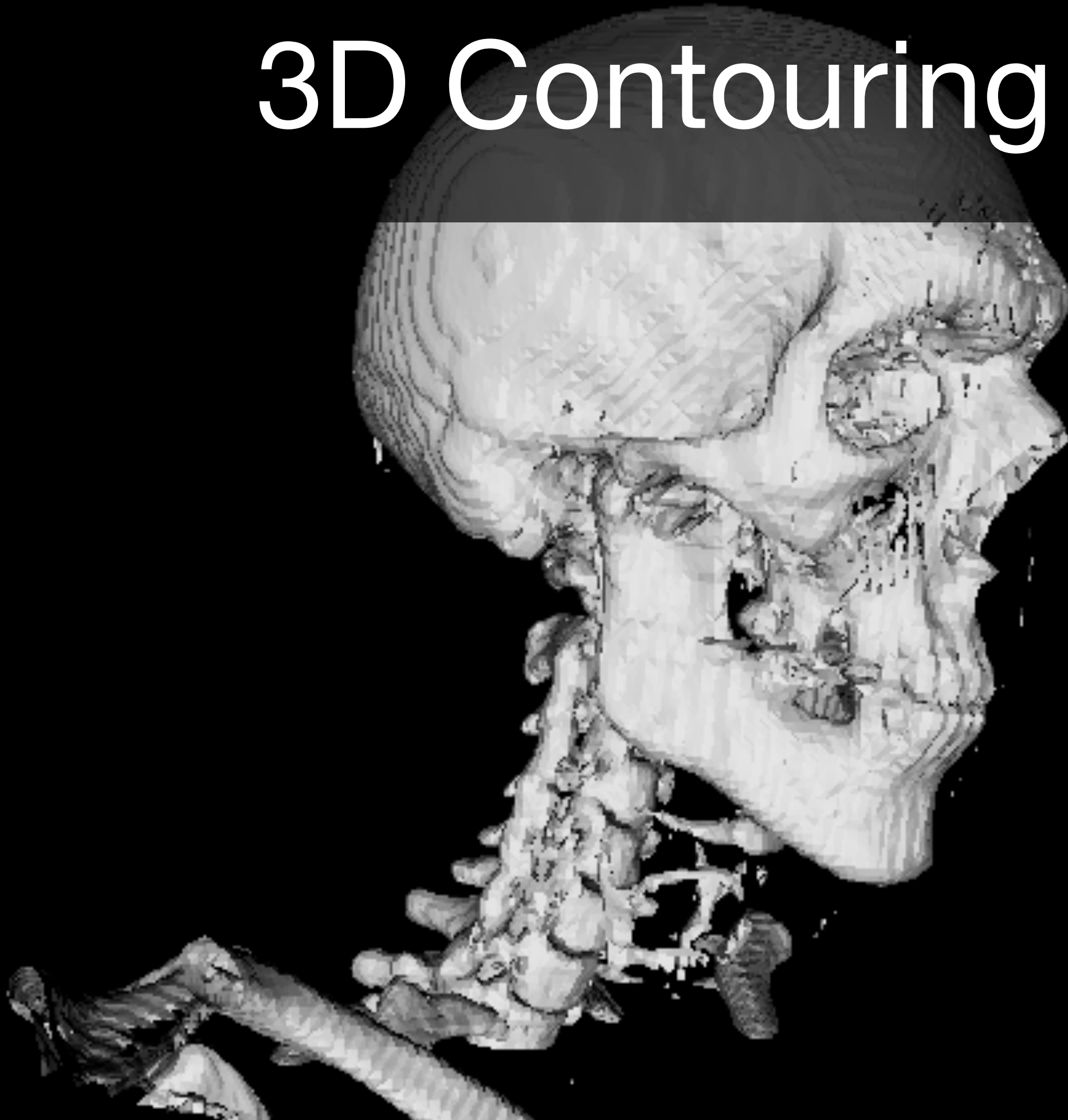


# Recap: 2D contouring



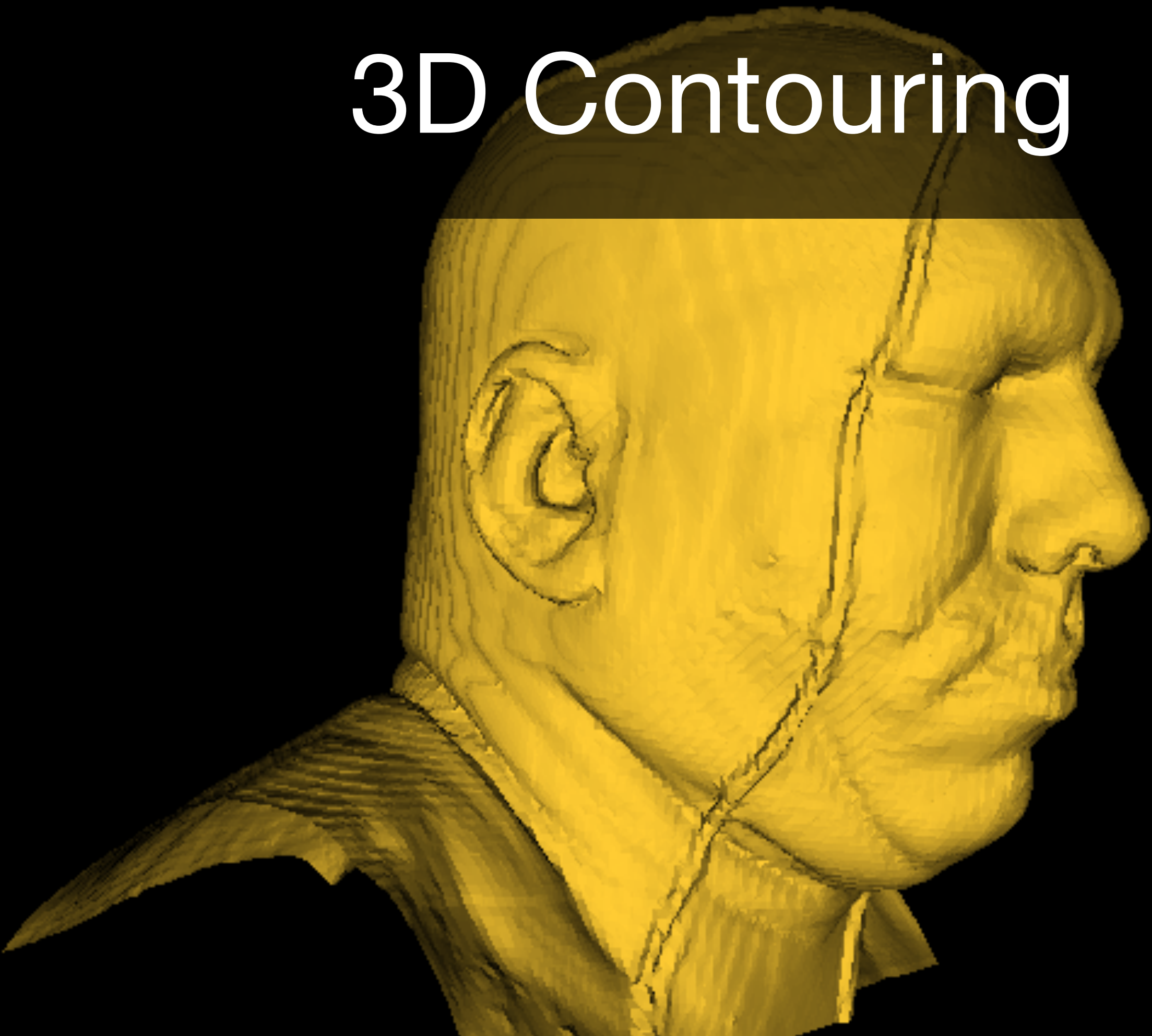
Case	Polarity	Rotation	Total	
No Crossings	x2		2	
Singlet	x2	x4	8	 <span>(x2 for polarity)</span>
Double adjacent	x2	x2 (4)	4	
Double Opposite	x2	x1 (2)	2	
			16 = 2 <sup>4</sup>	

# 3D Contouring

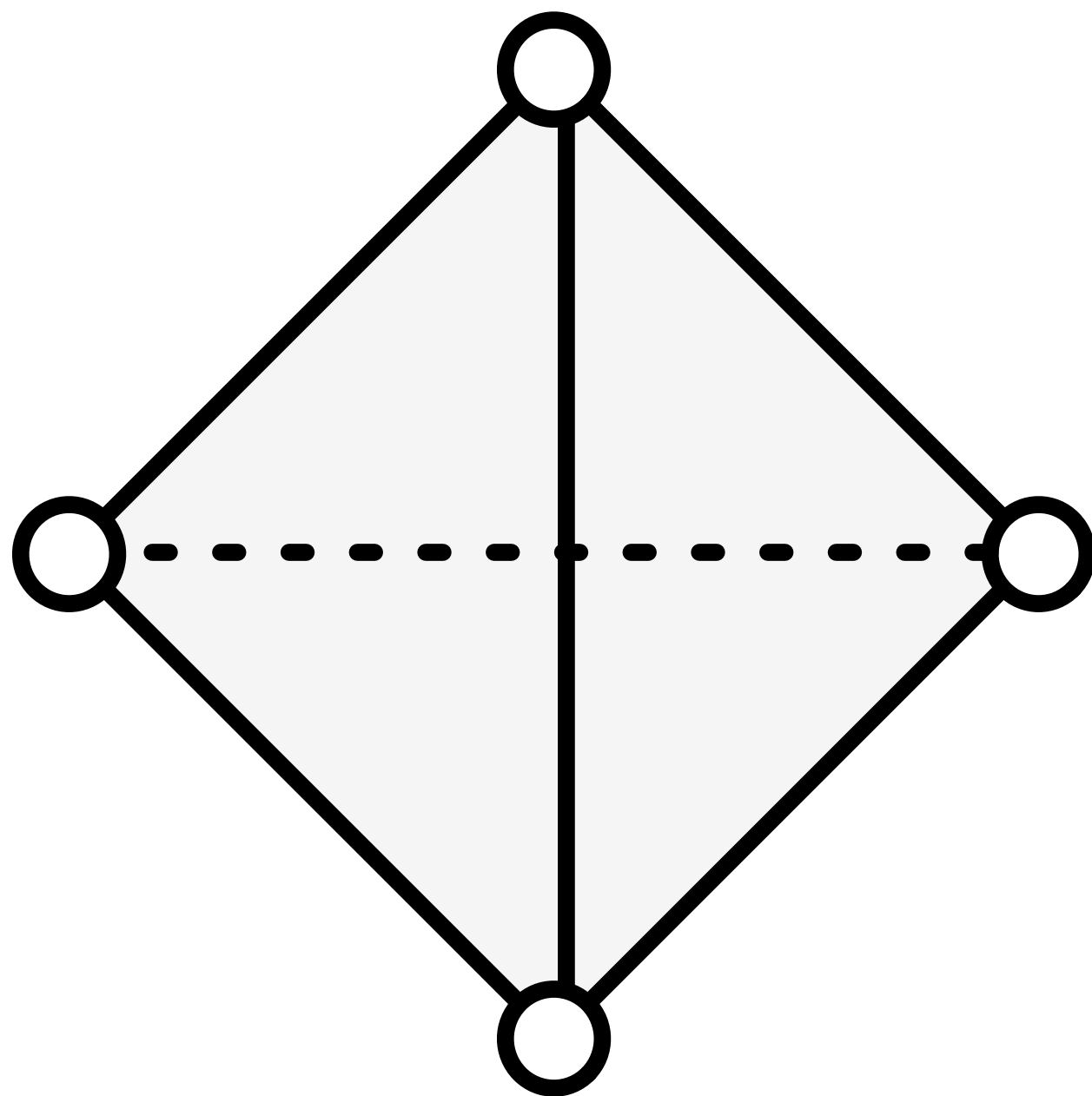
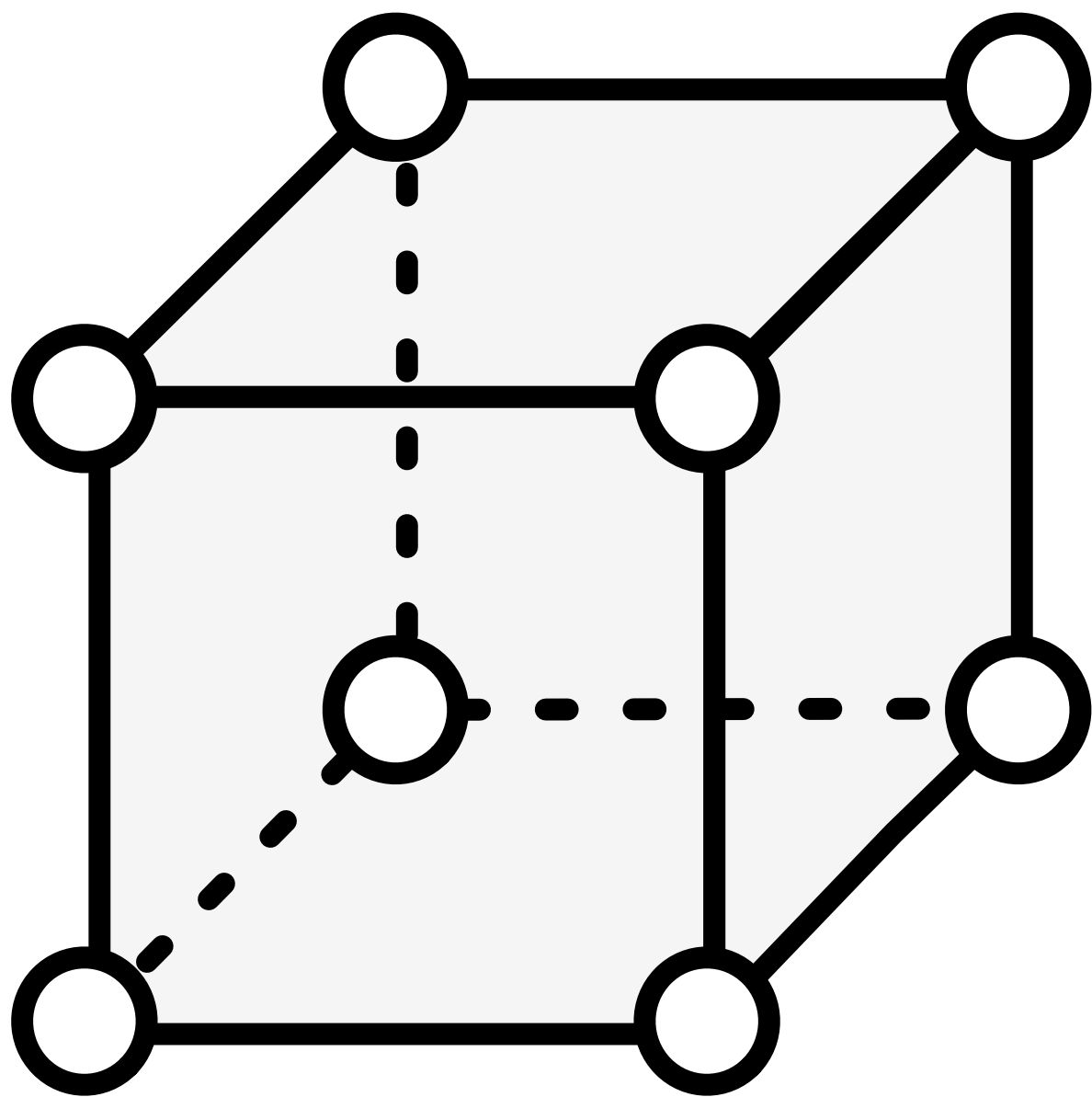




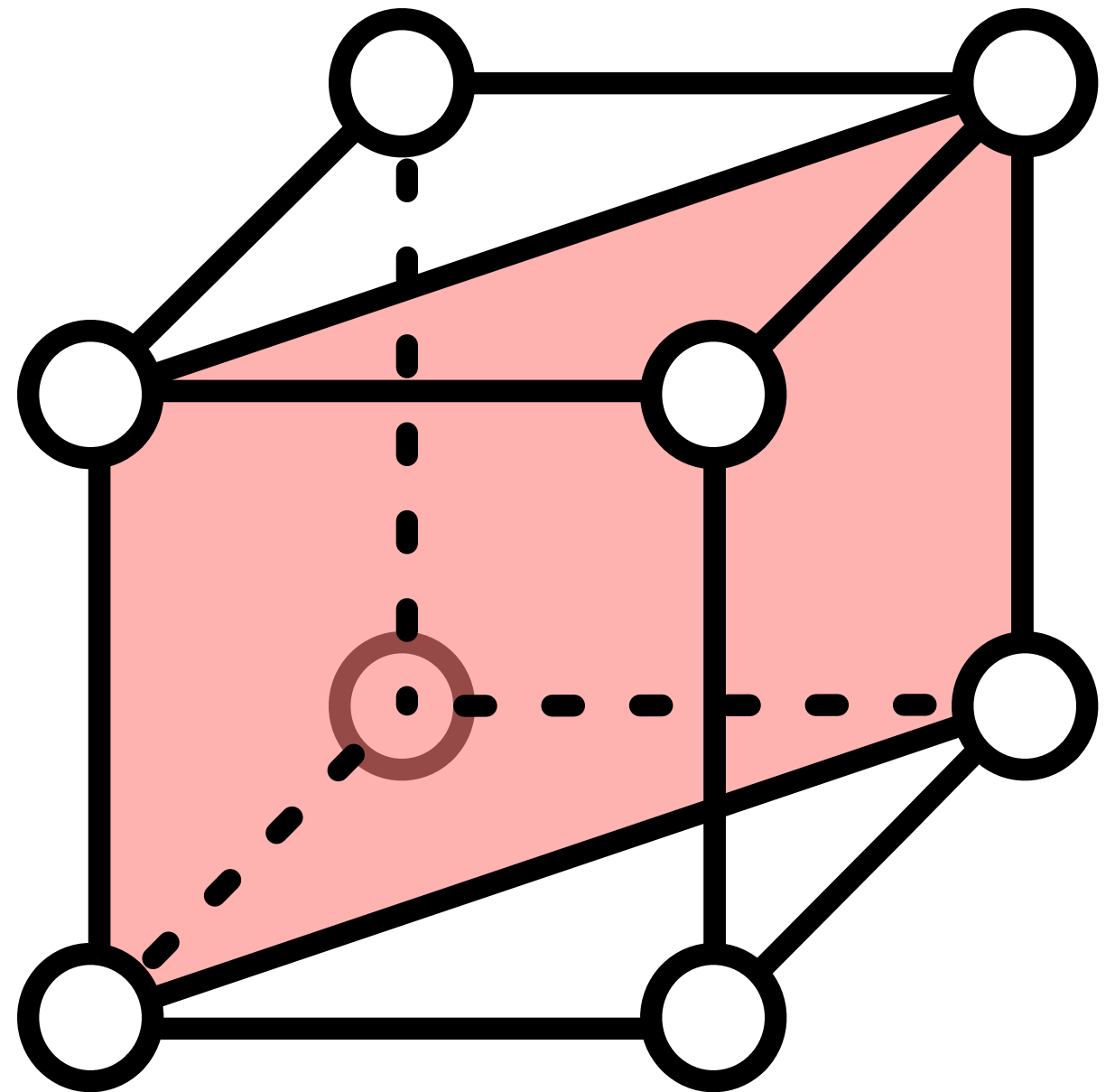
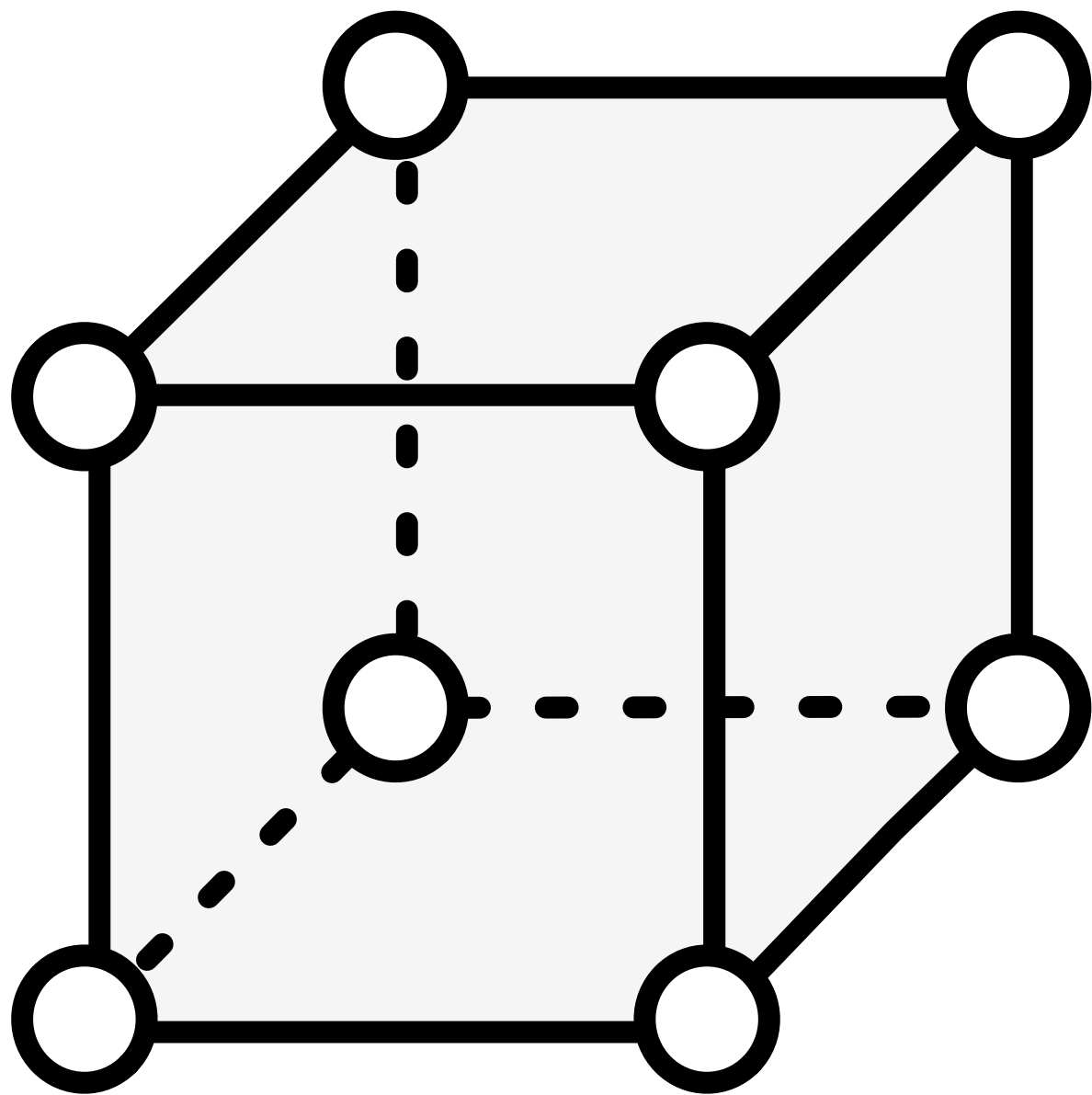
# 3D Contouring



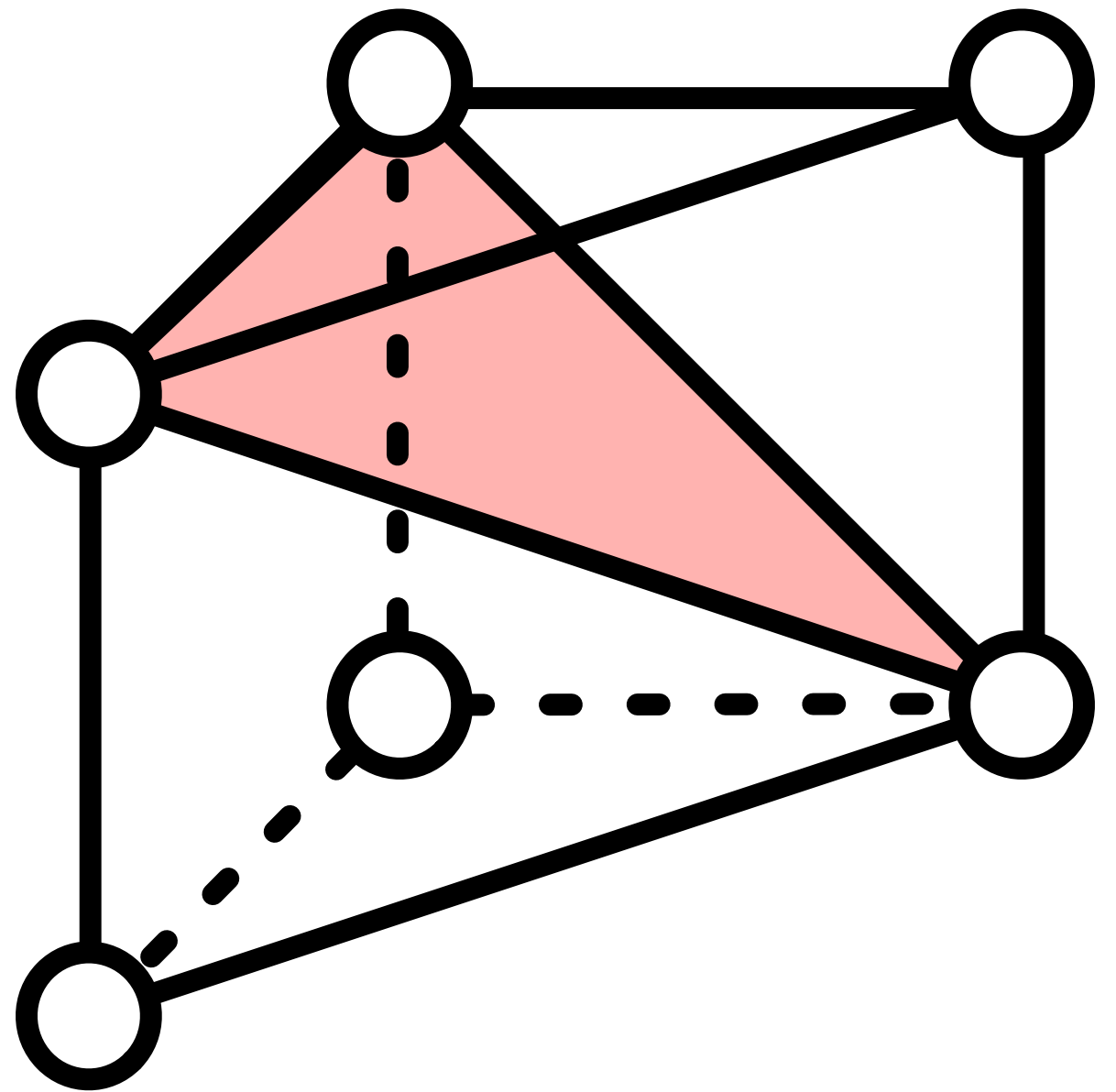
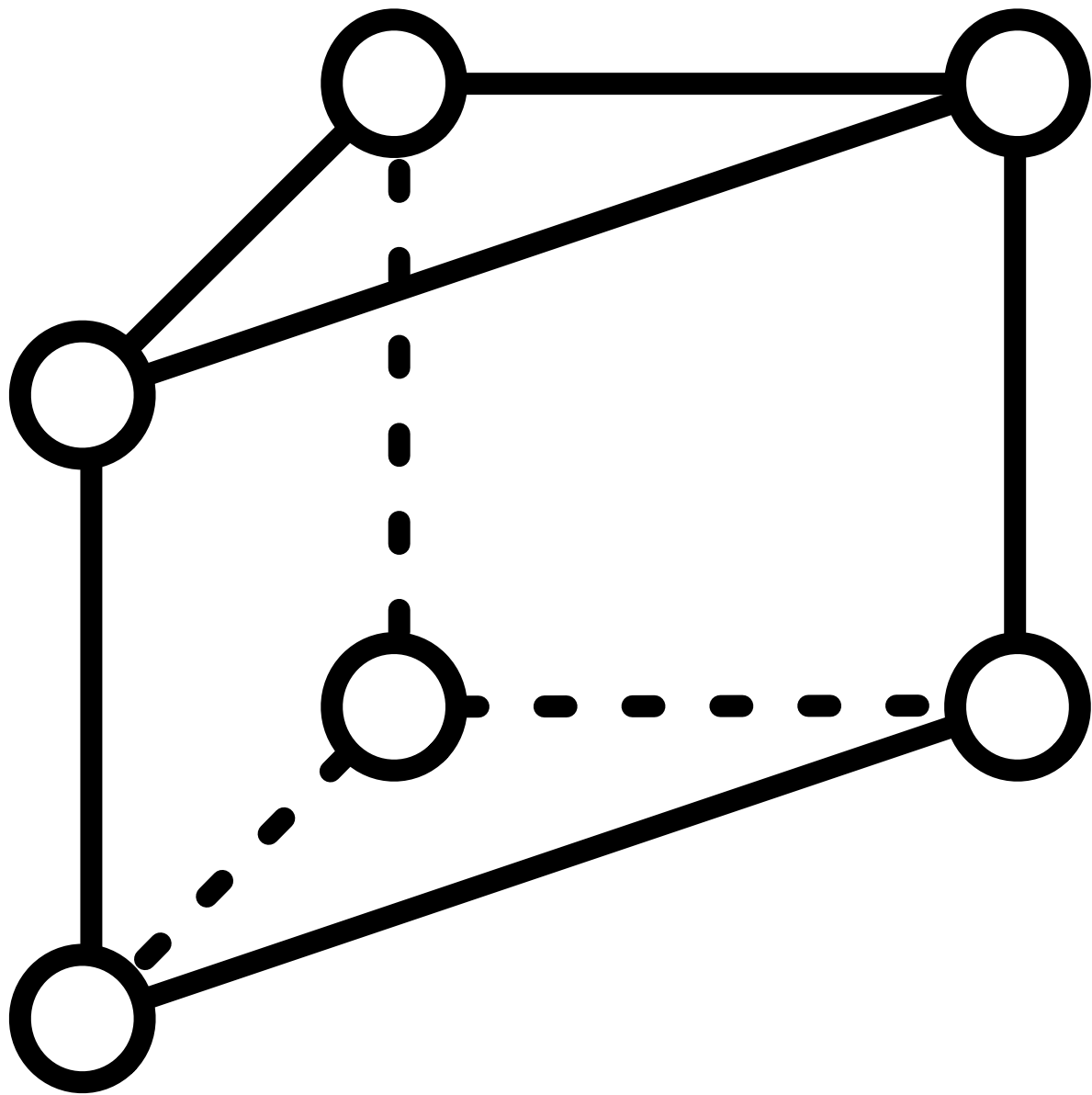
# Splitting 3D space into simple shapes



# Cube into tetrahedra

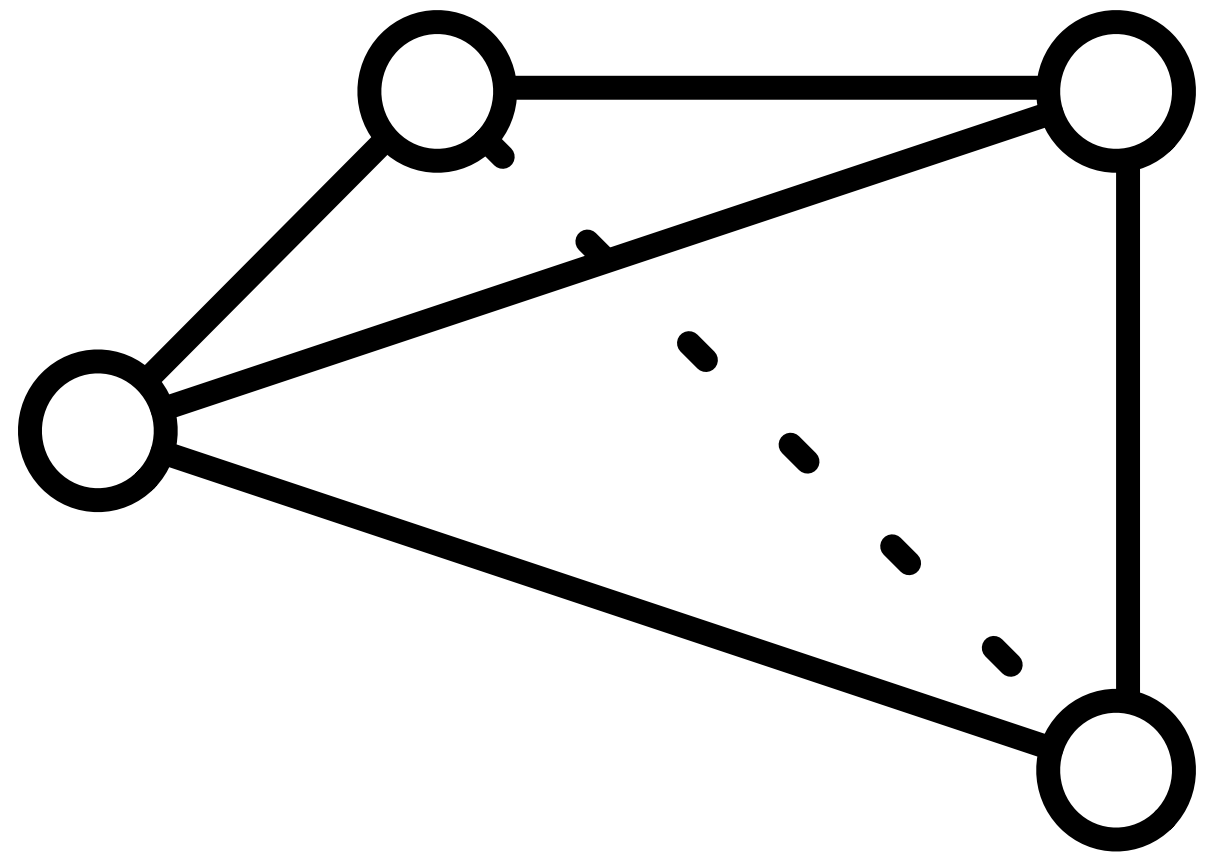
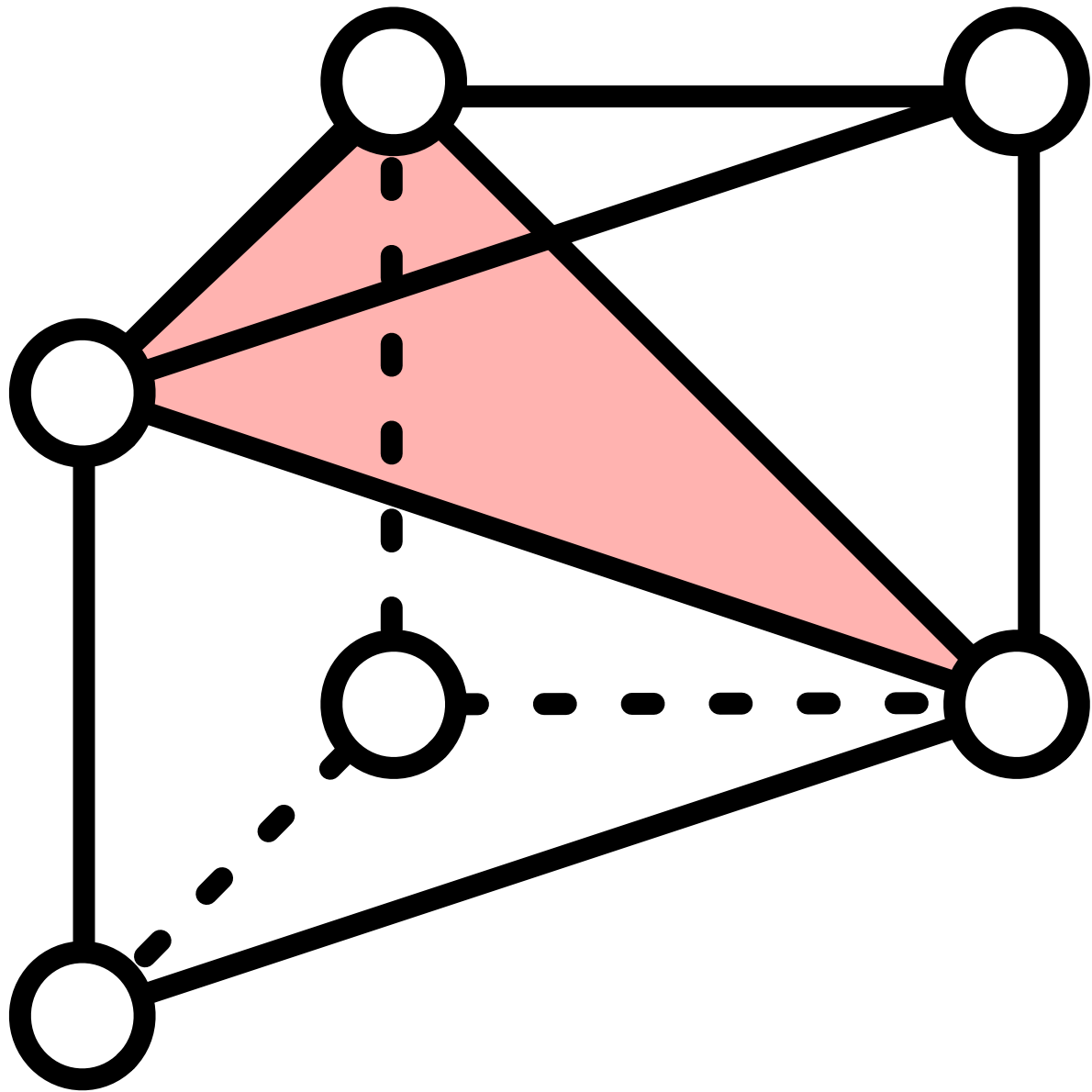


# Cube into tetrahedra



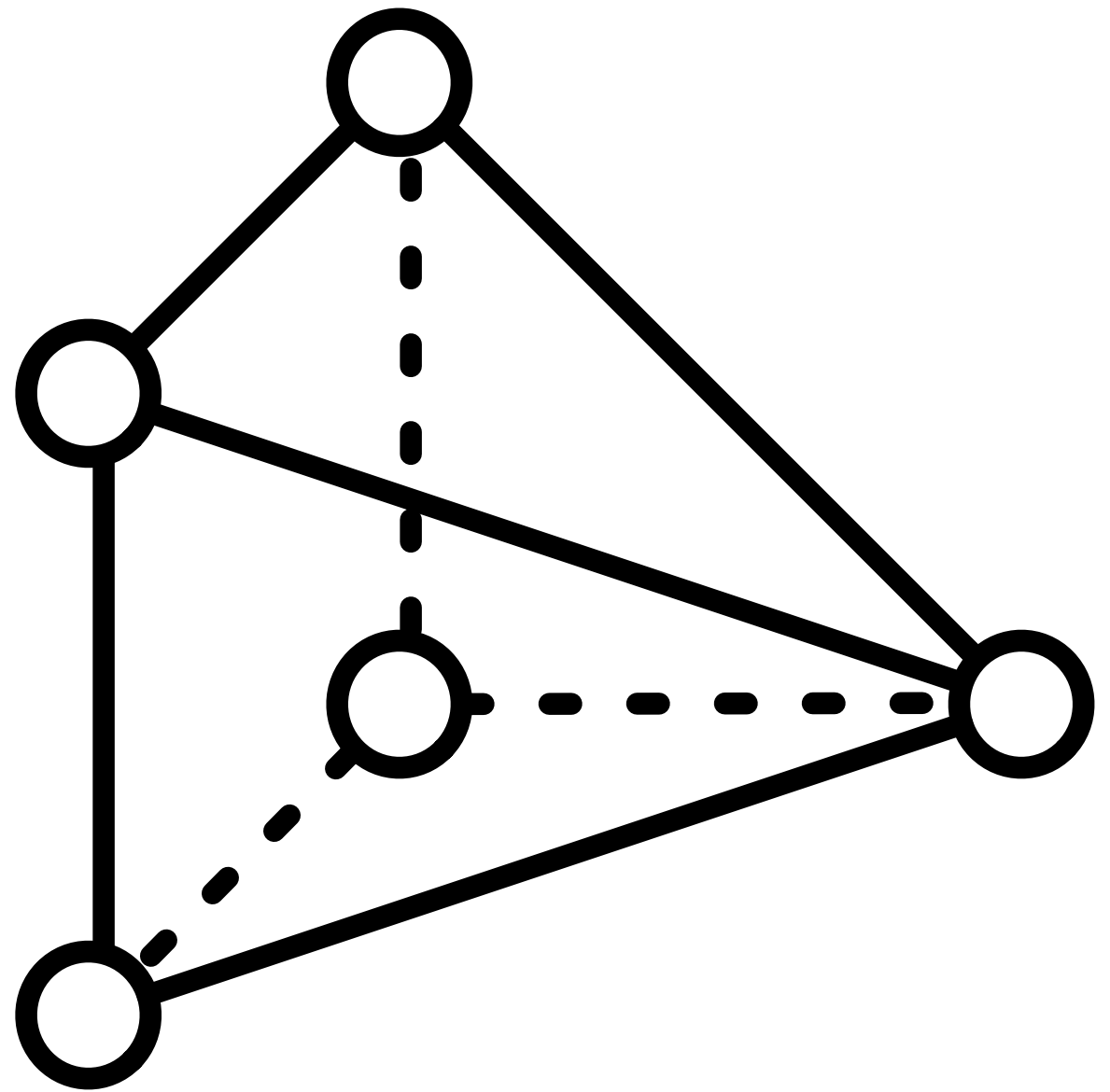
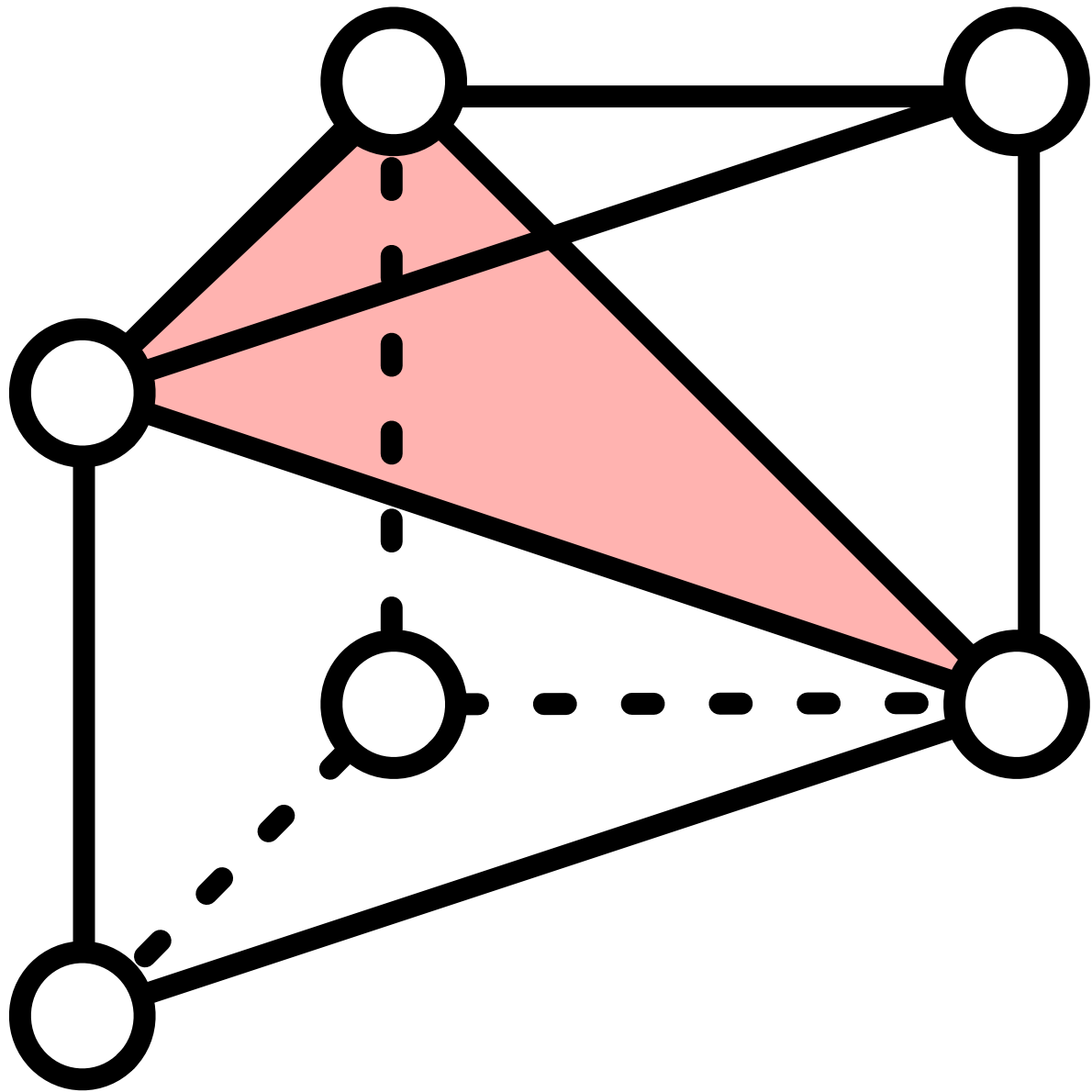


# Cube into tetrahedra

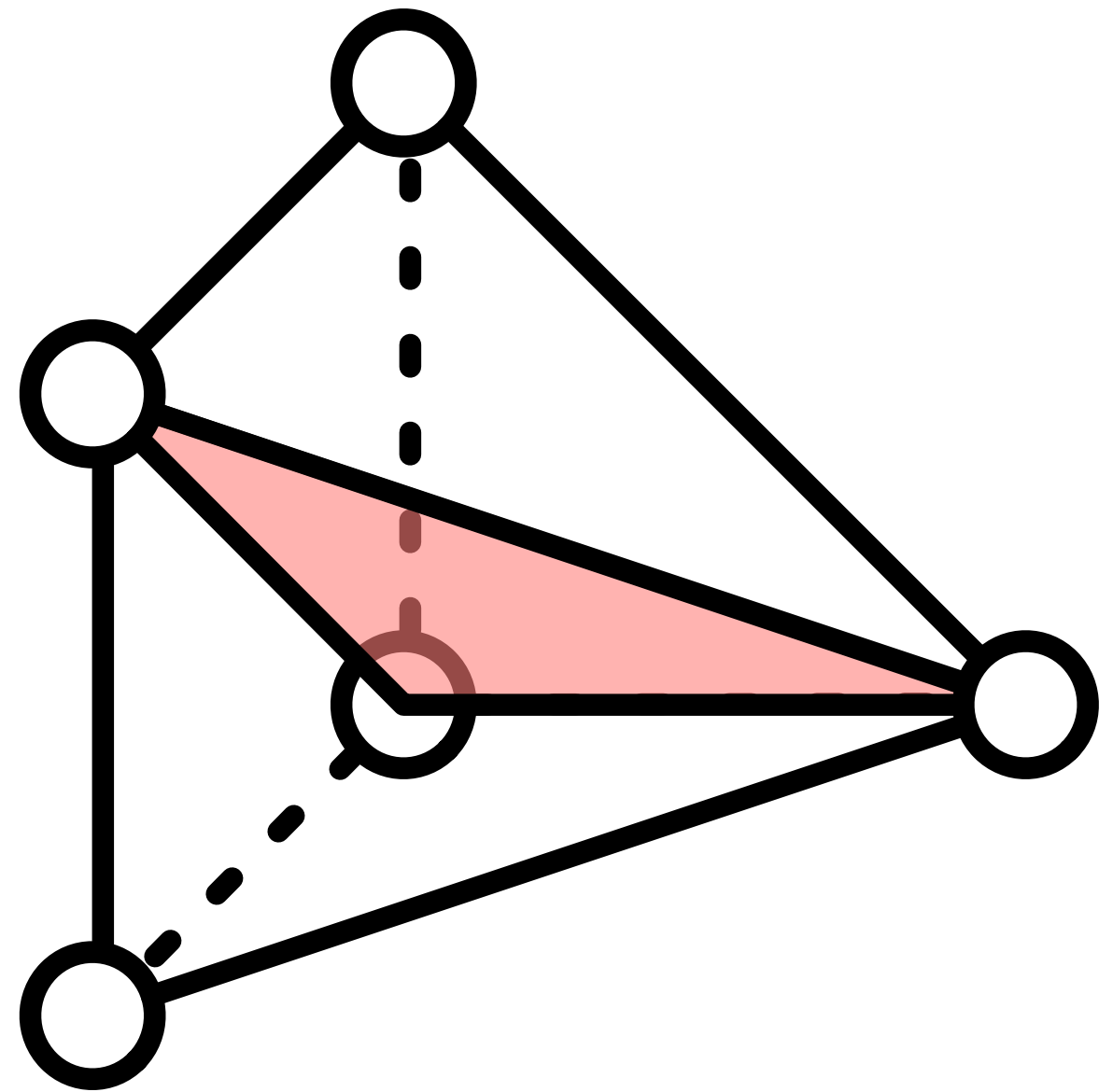
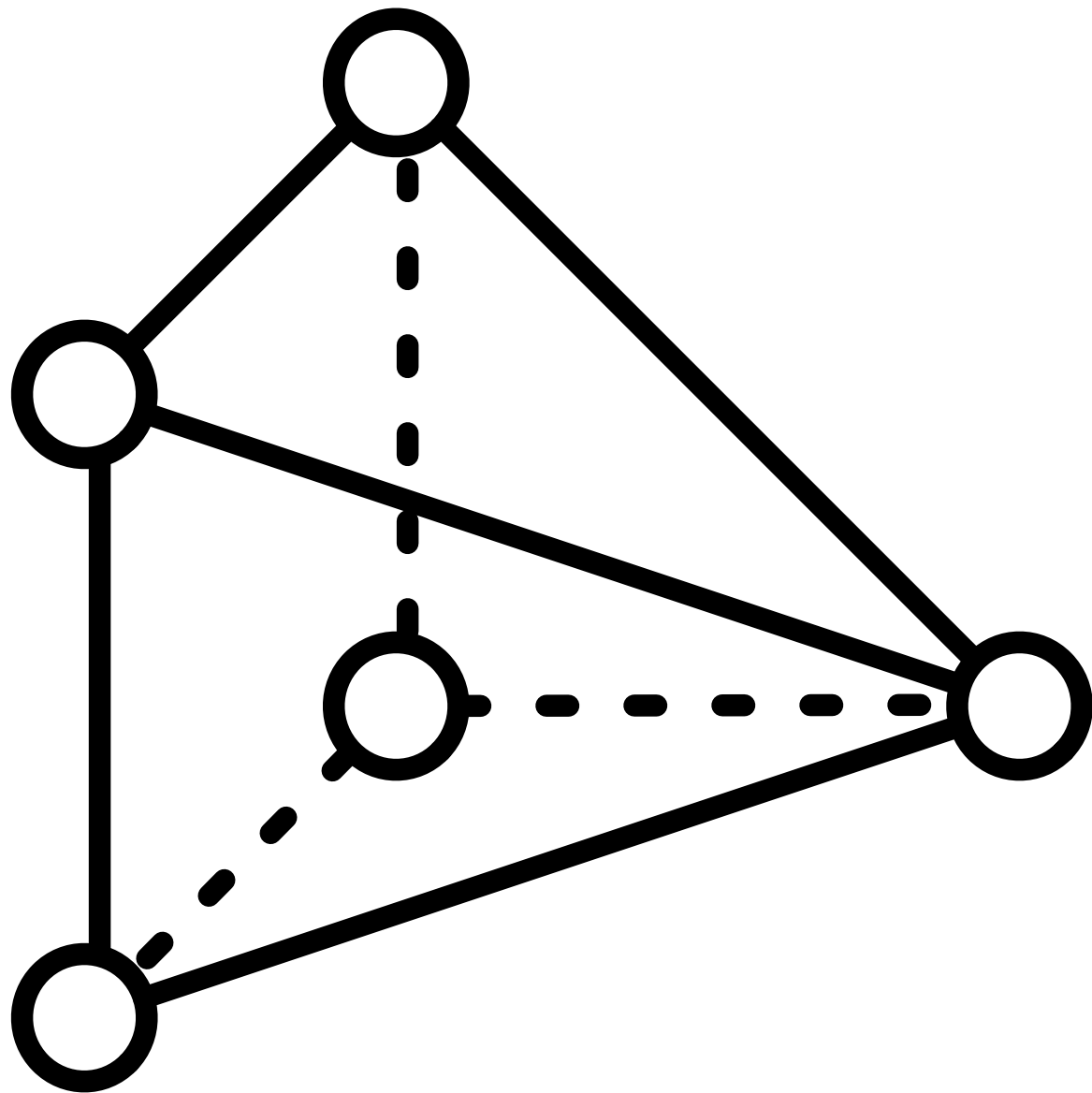


1 tetrahedron,

# Cube into tetrahedra

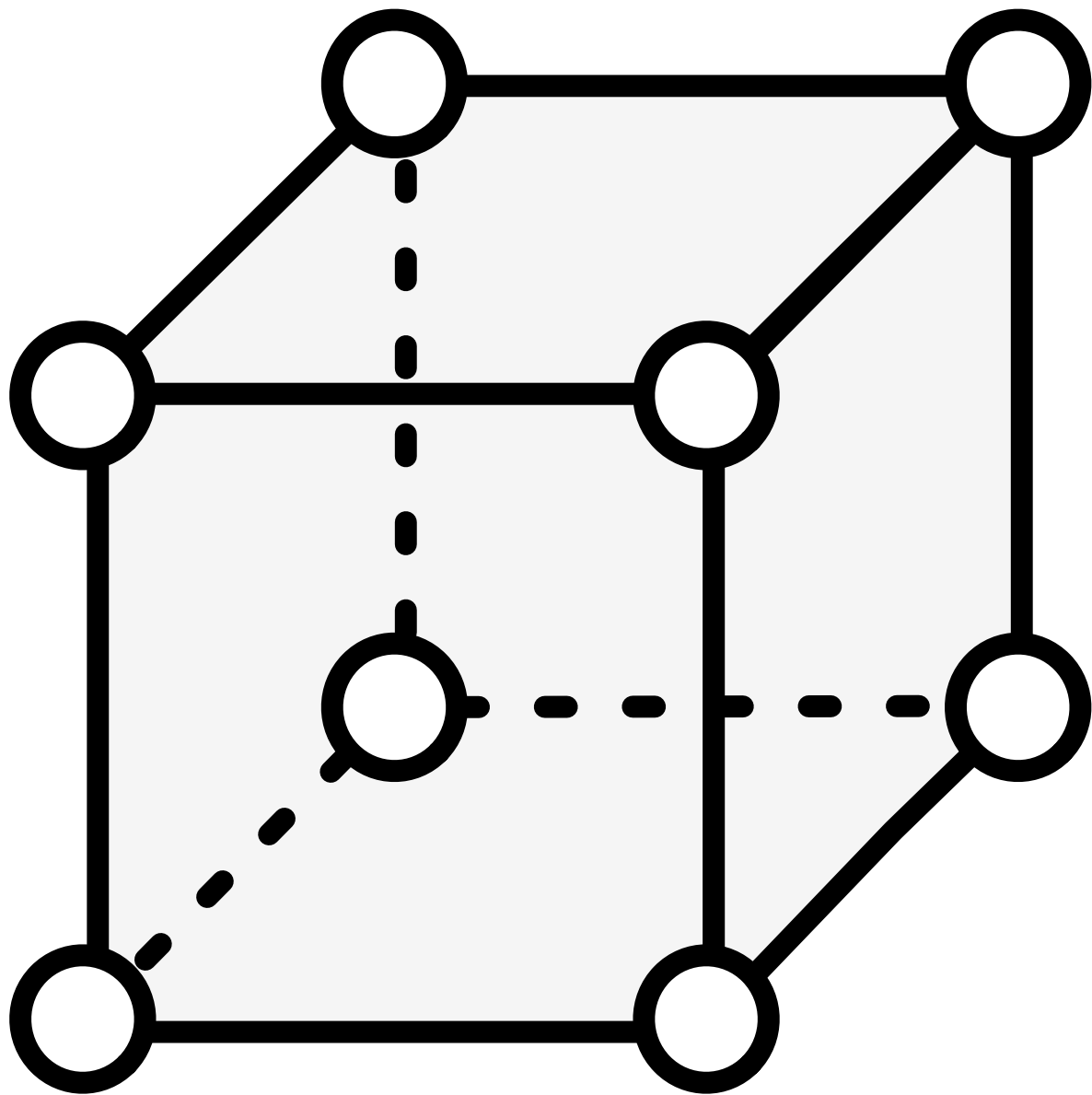


# Cube into tetrahedra



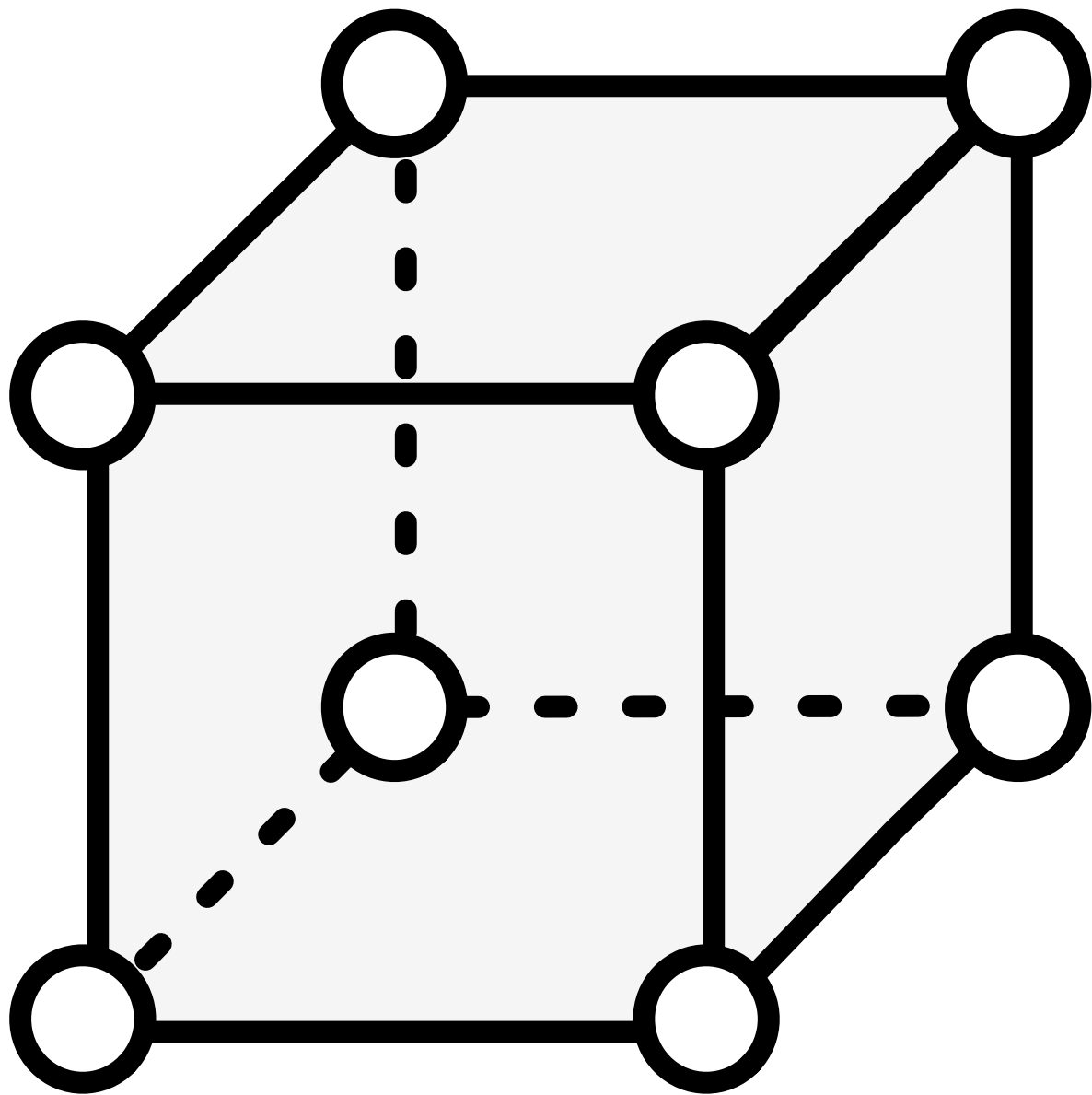
2 tetrahedra

# Cube into tetrahedra



1 cube splits into  
6 tetrahedra

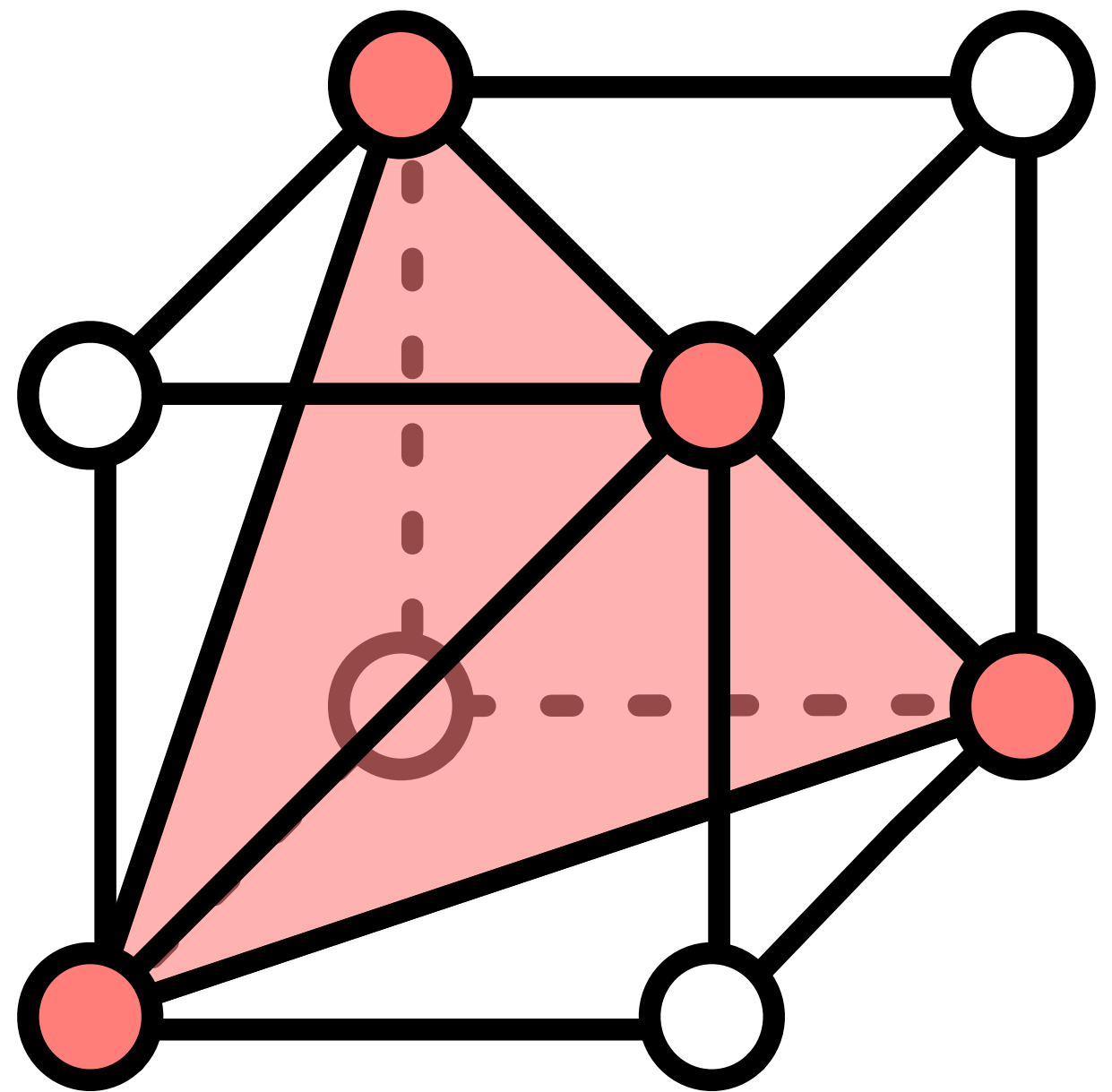
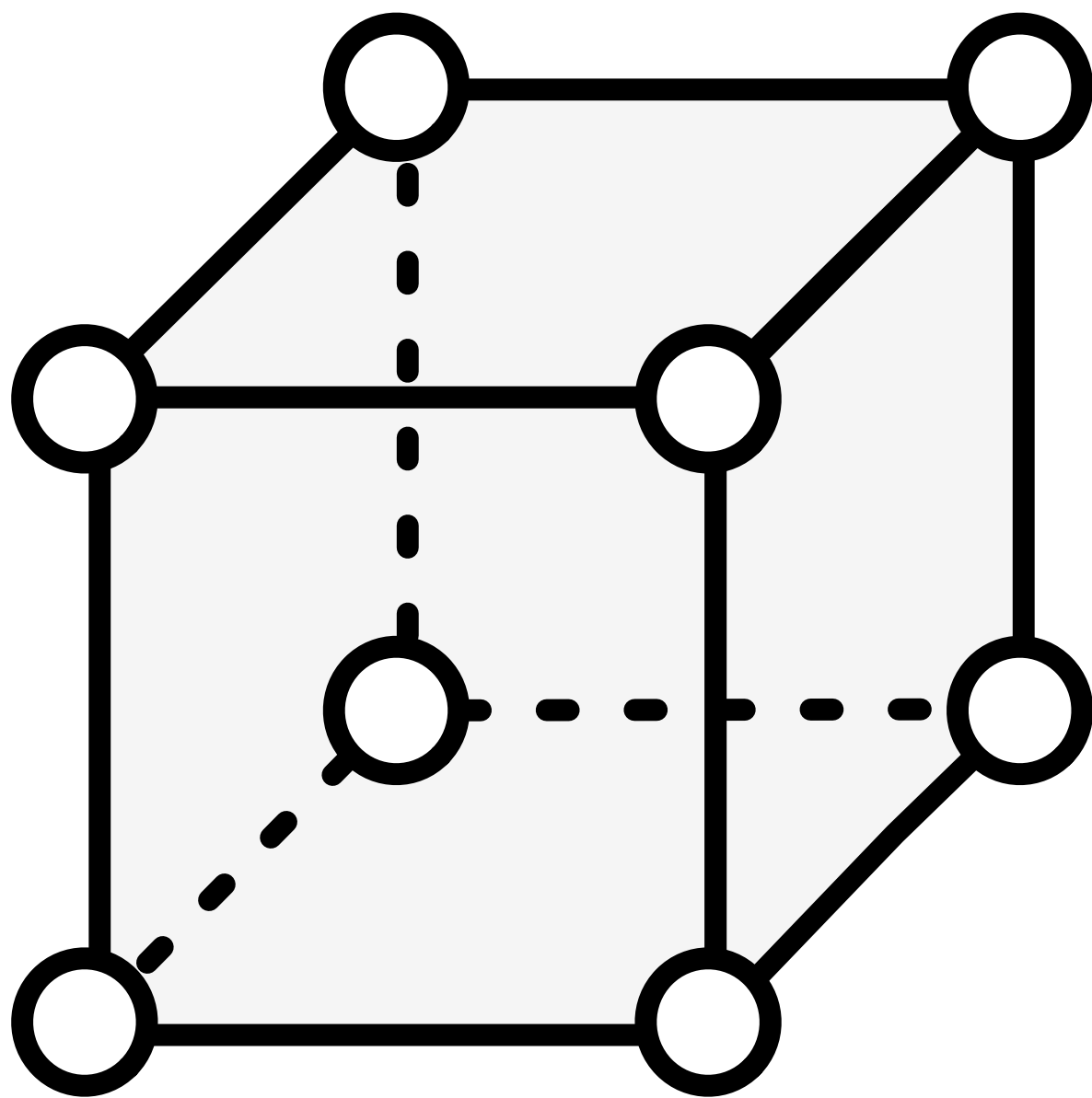
# Cube into tetrahedra



1 cube splits into  
6 tetrahedra...

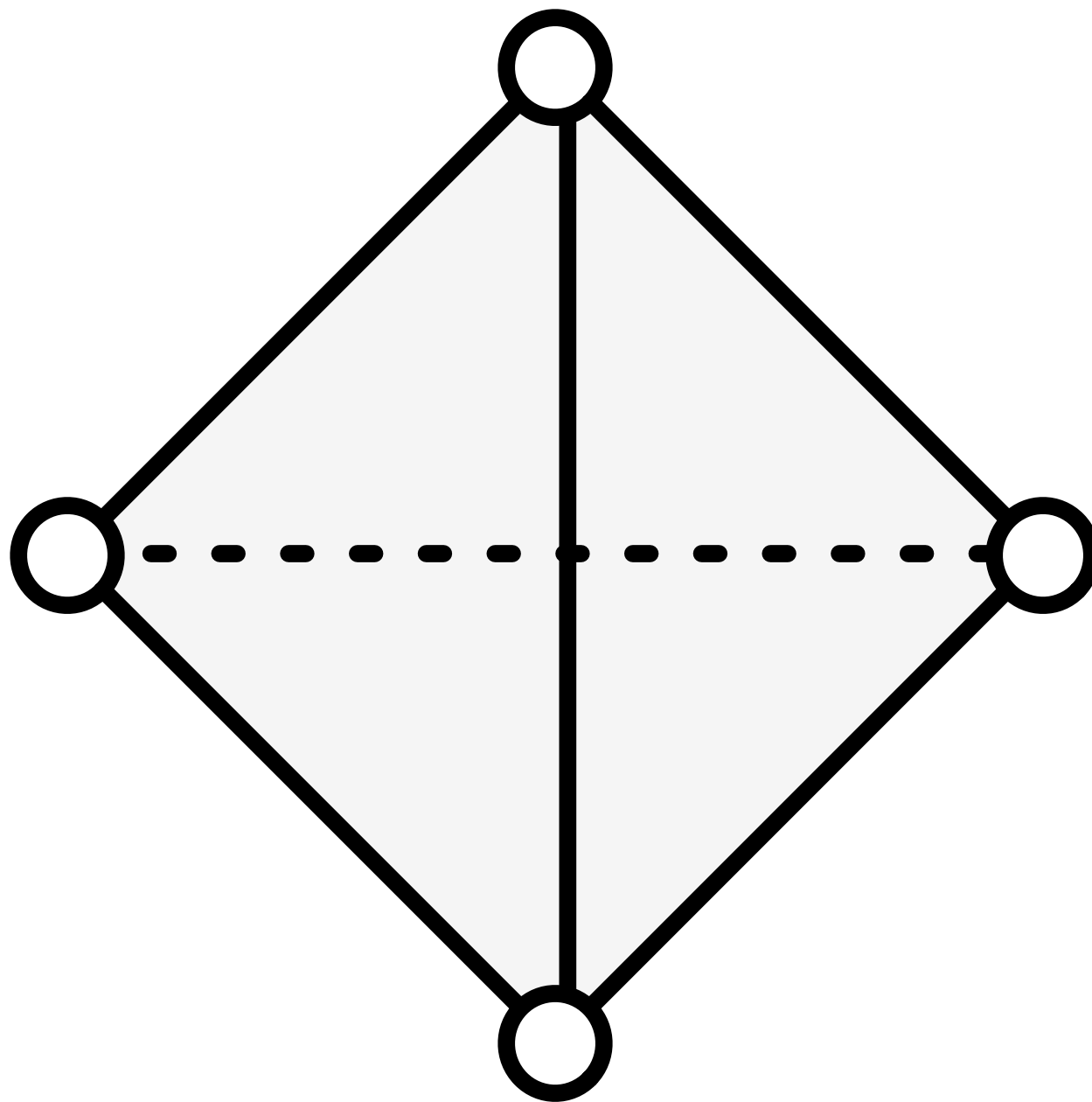
**but also into 5 tetrahedra!**

# Cube into tetrahedra

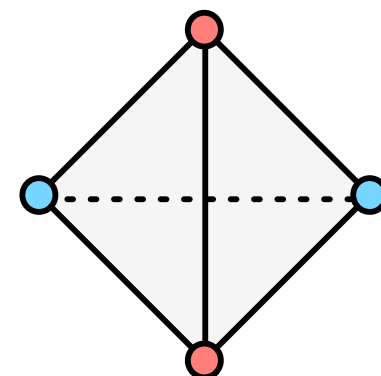
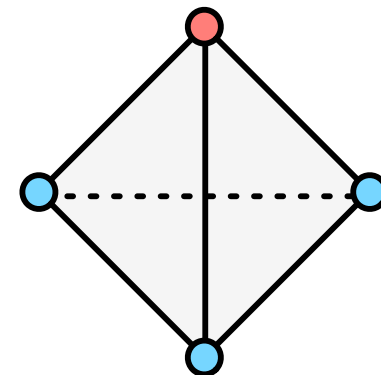
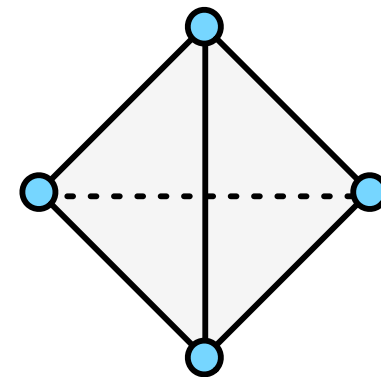




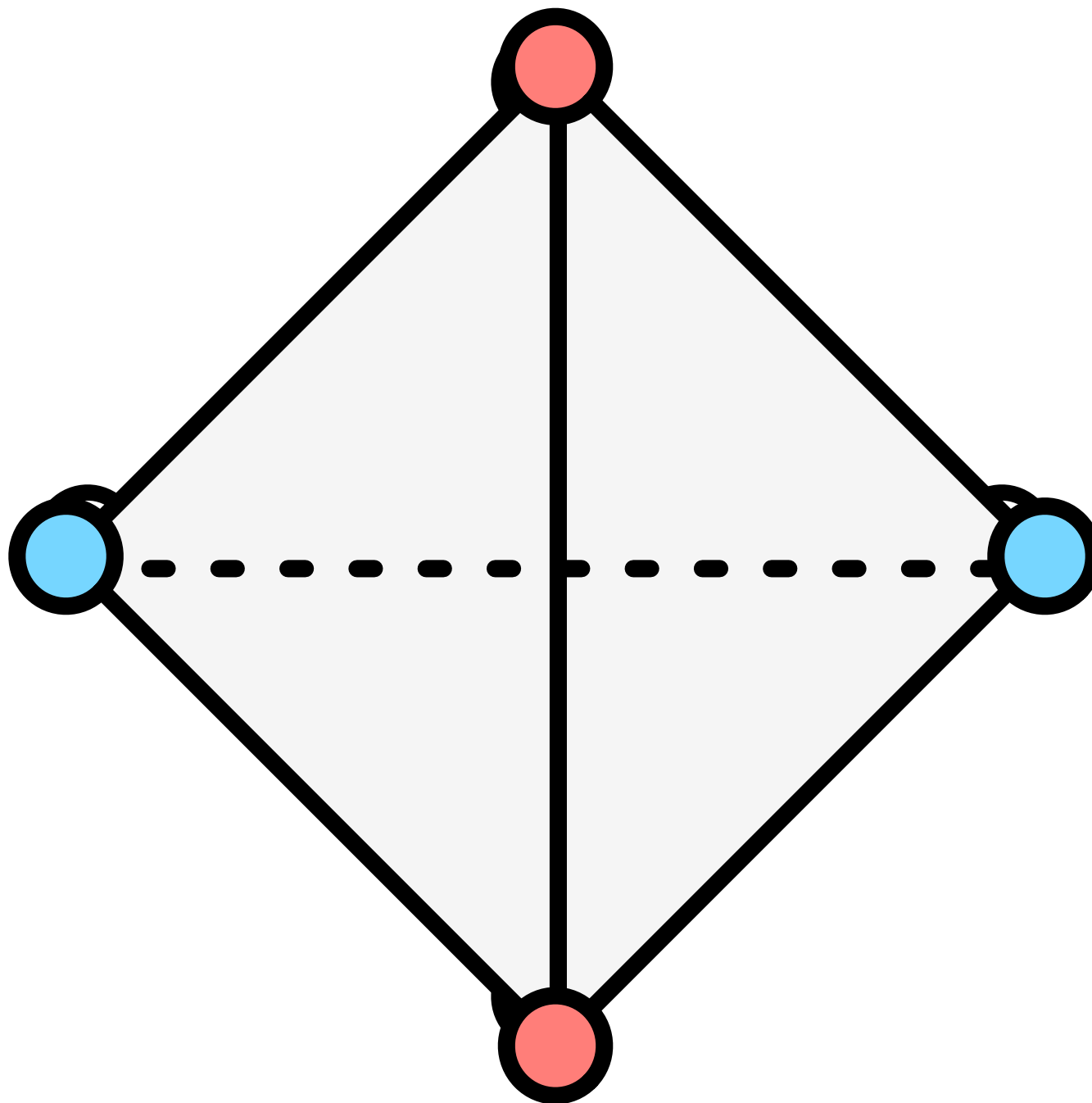
# Marching Tetrahedra



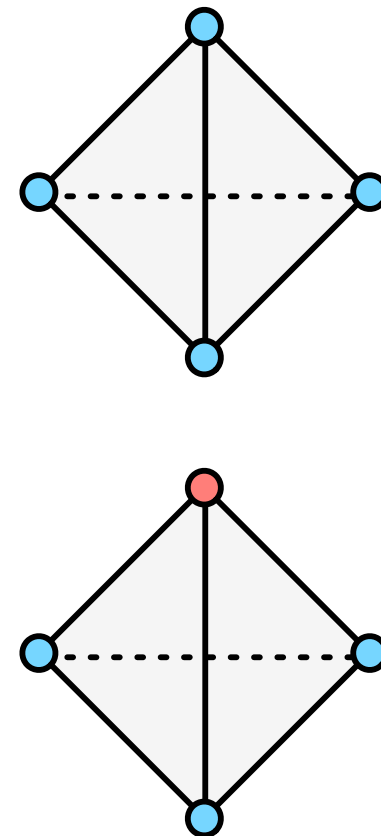
3 cases, “obvious”



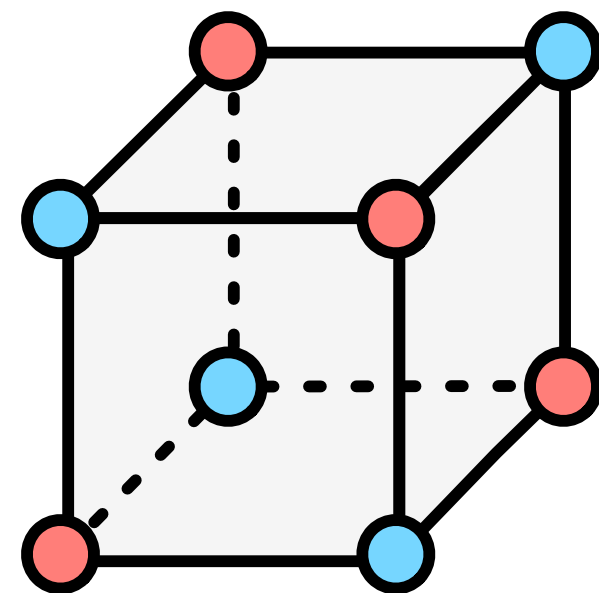
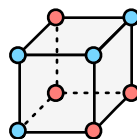
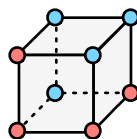
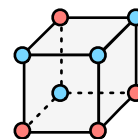
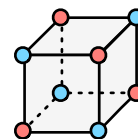
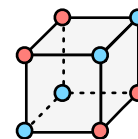
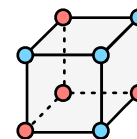
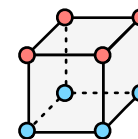
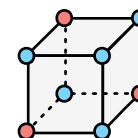
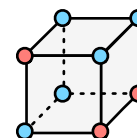
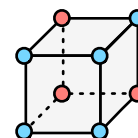
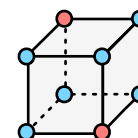
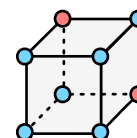
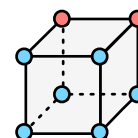
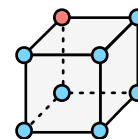
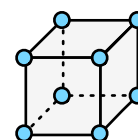
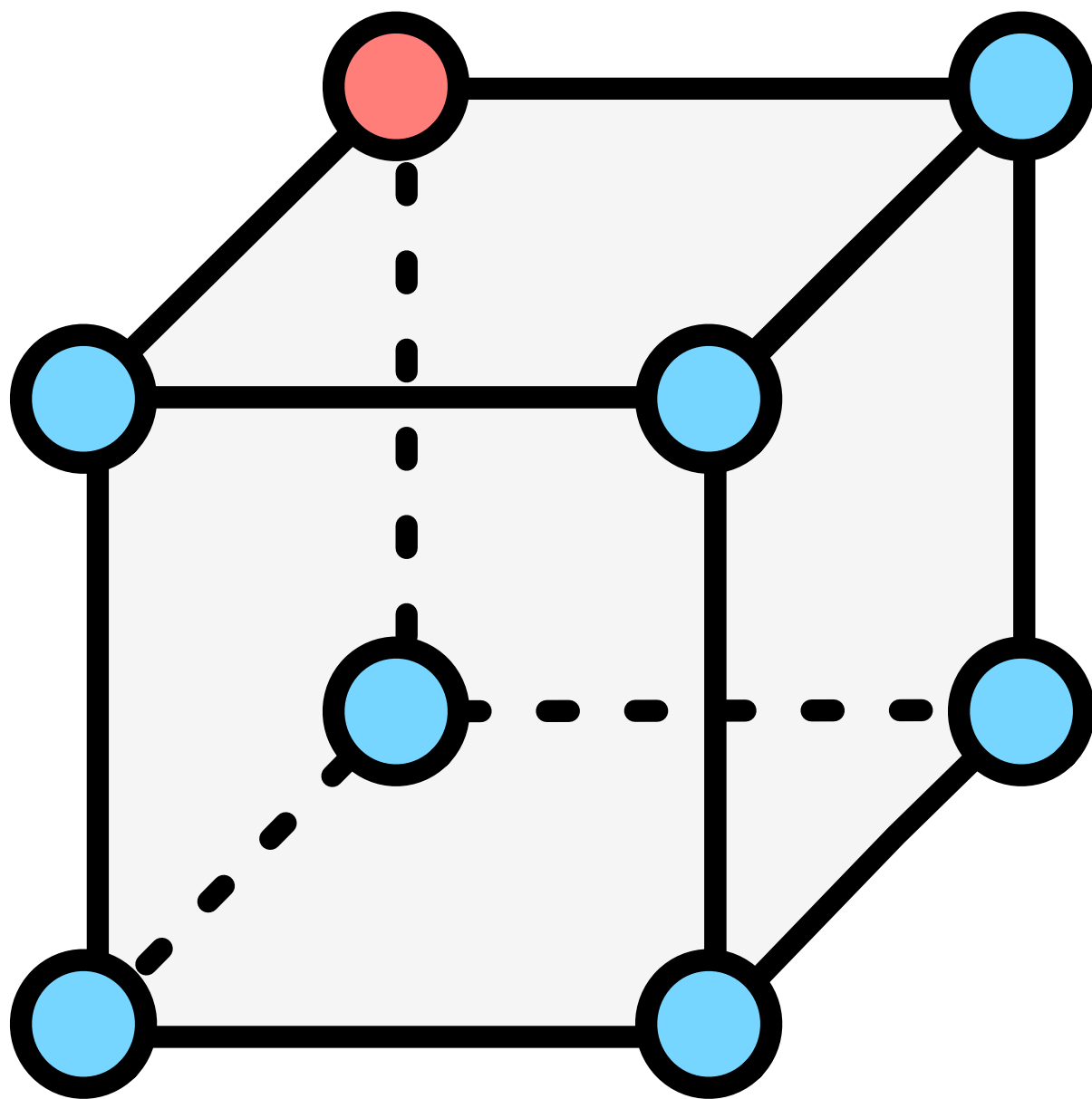
# Marching Tetrahedra



3 cases, “obvious”



# 3D Contouring



# 3D Contouring



Computer Graphics, Volume 21, Number 4, July 1987

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## MARCHING CUBES: A HIGH RESOLUTION 3D SURFACE CONSTRUCTION ALGORITHM

*William E. Lorensen*  
*Harvey E. Cline*

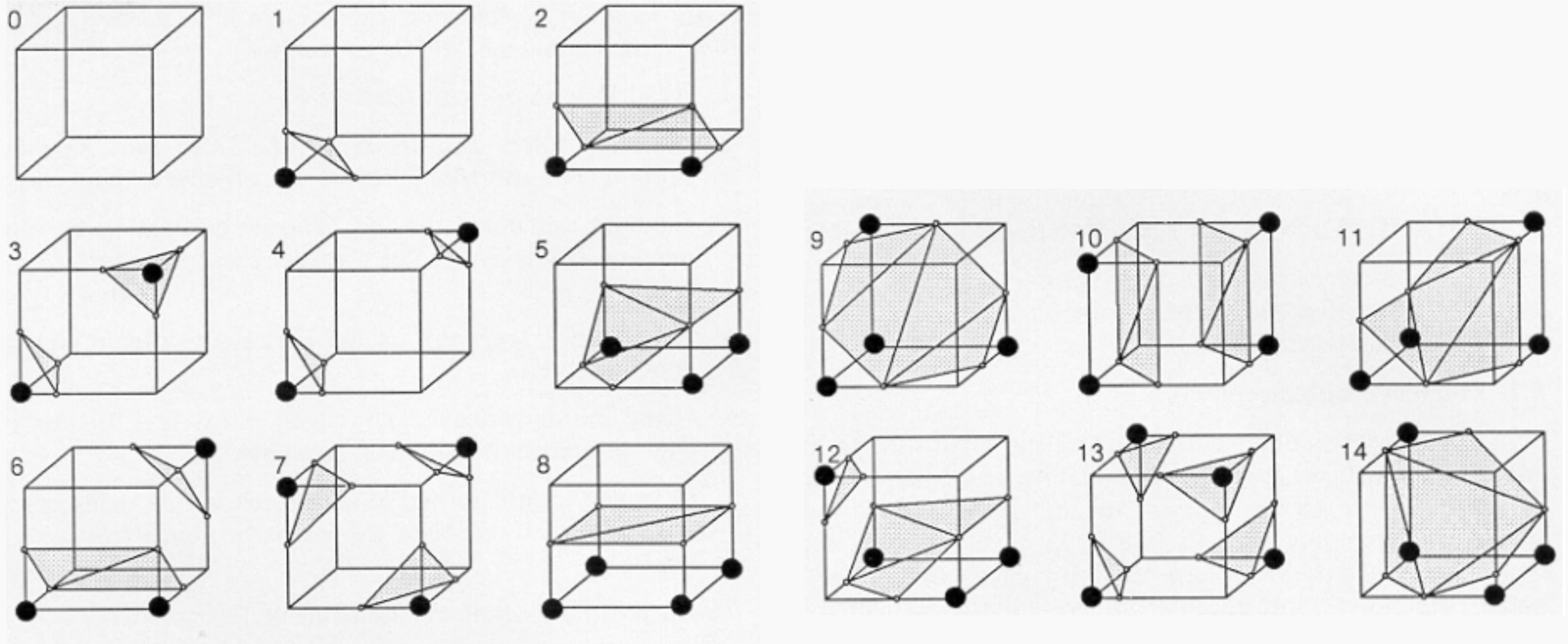
General Electric Company  
Corporate Research and Development  
Schenectady, New York 12301

### Abstract

We present a new algorithm, called *marching cubes*, that creates triangle models of constant density surfaces from 3D medical data. Using a divide-and-conquer approach to generate inter-slice connectivity, we create a case table that defines triangle topology. The algorithm processes the 3D medical data in scan-line order and calculates triangle vertices using linear interpolation. We find the gradient of the origi-

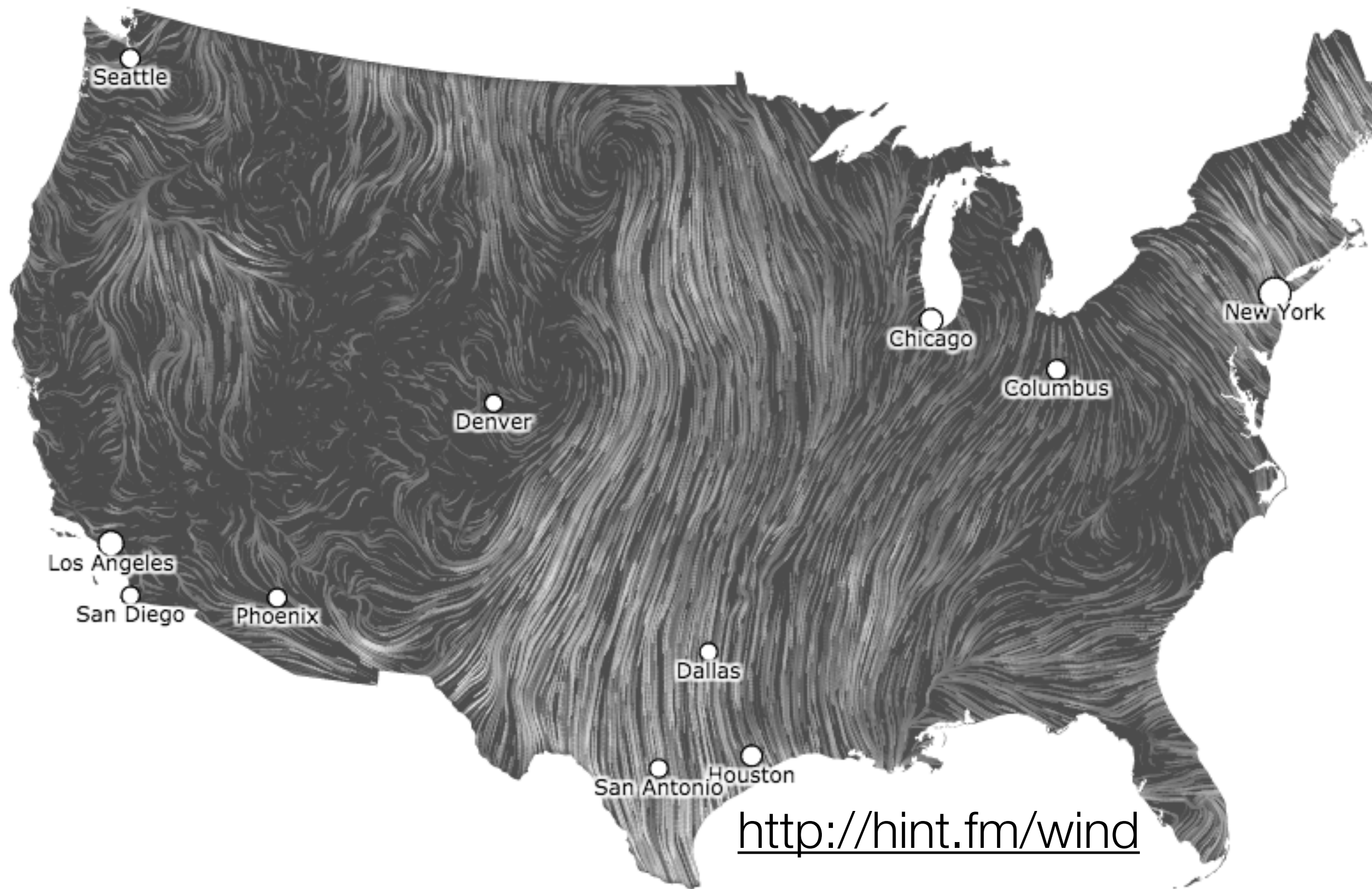
acetabular fractures [6], craniofacial abnormalities [17,18], and intracranial structure [13] illustrate 3D's potential for the study of complex bone structures. Applications in radiation therapy [27,11] and surgical planning [4,5,31] show interactive 3D techniques combined with 3D surface images. Cardiac applications include artery visualization [2,16] and non-graphic modeling applications to calculate surface area and volume [21].

# 3D Contouring



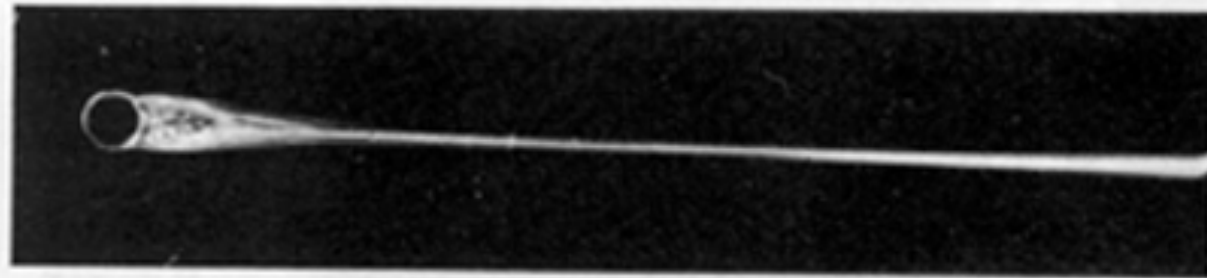






# Spatial Data: Vector Fields

# Experimental Flow Vis



$R = 32$



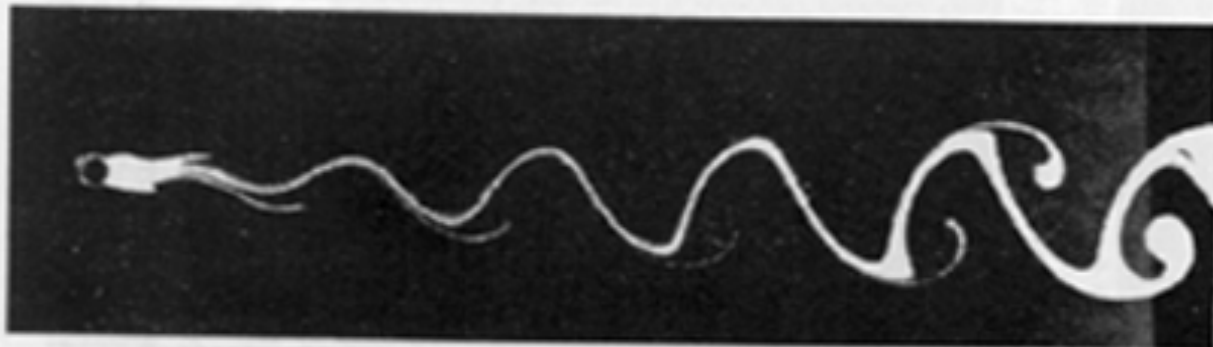
$R = 73$



$R = 55$



$R = 102$



$R = 65$



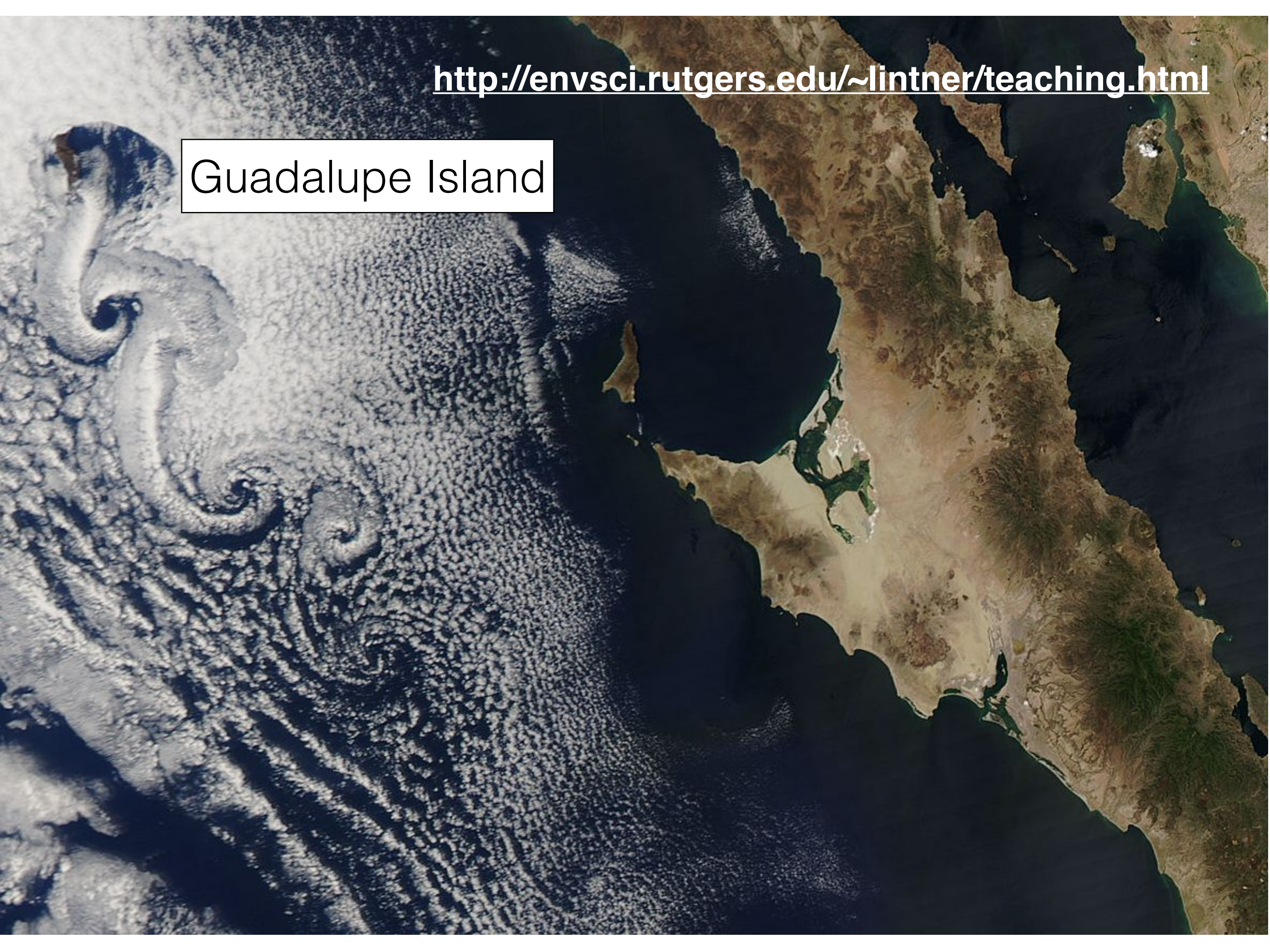
$R = 161$

von Kármán vortex street, depending on Reynolds number



<http://envsci.rutgers.edu/~lintner/teaching.html>

Guadalupe Island





# Mathematics of Vector Fields

$$v : R^n \rightarrow R^n$$

**Function from vectors to vectors**

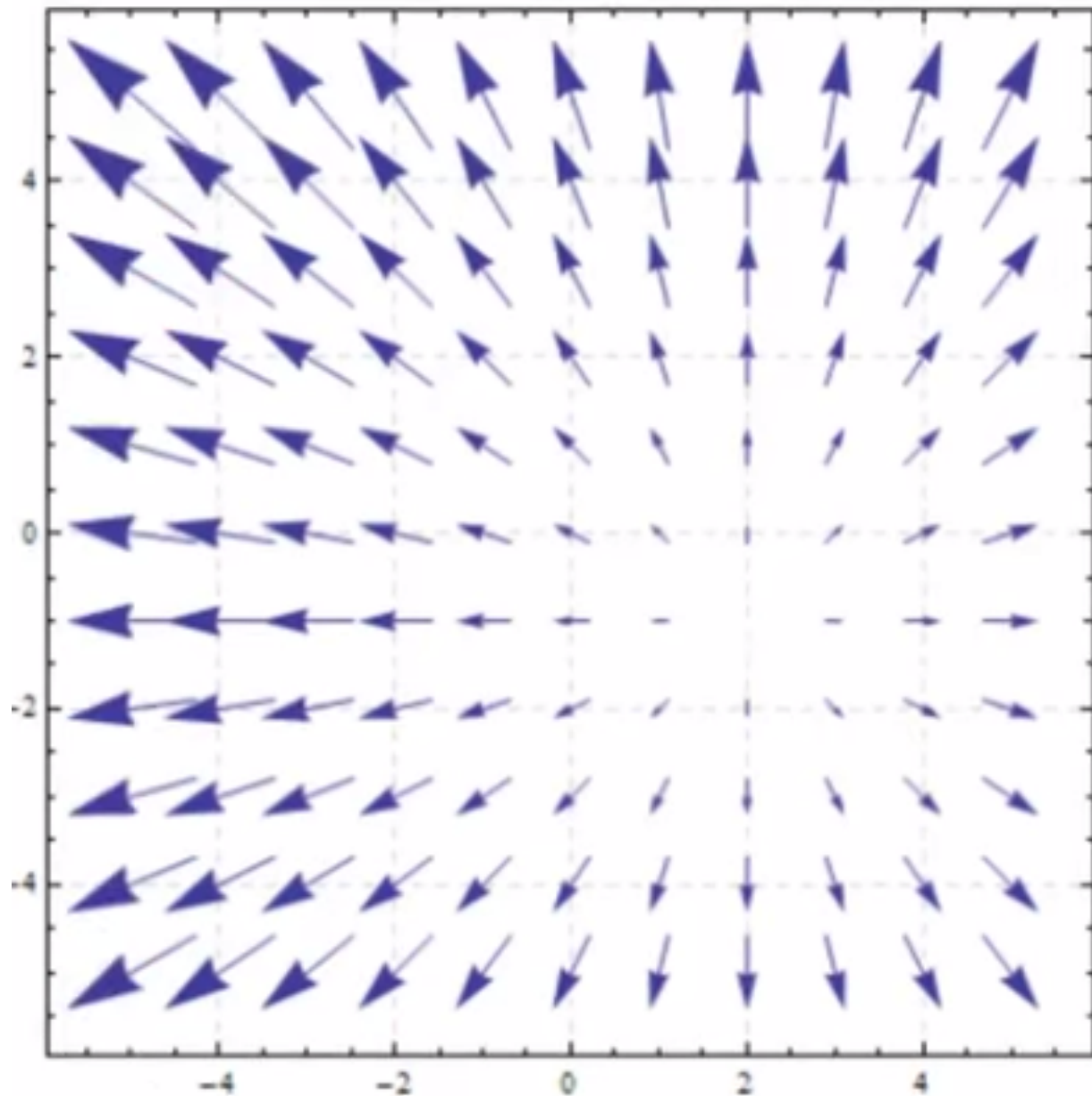
<https://www.youtube.com/watch?v=nuQyKGuXJOs>



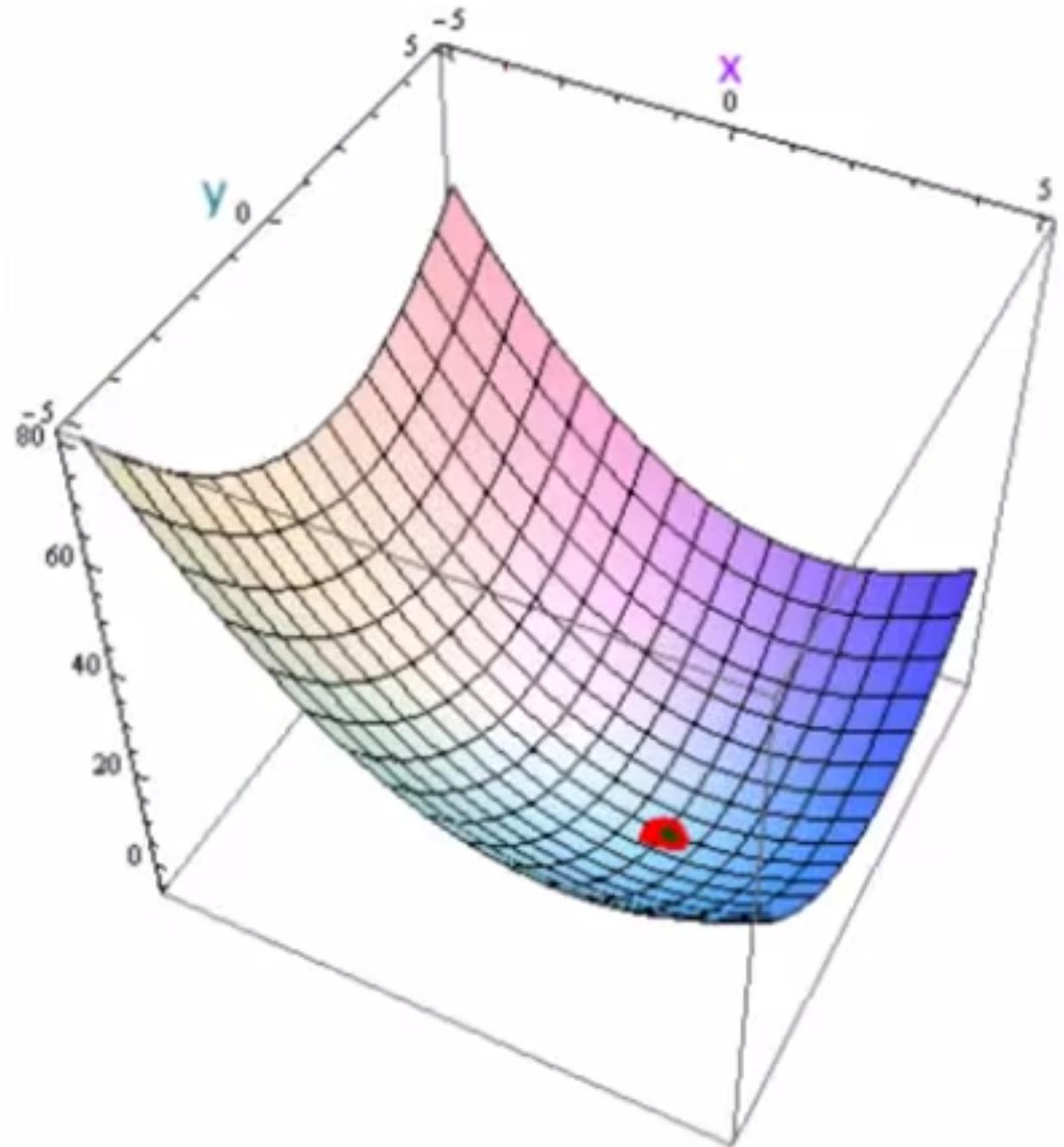
# Spatial Data: Vector Fields

# A simple vector field: the gradient

The gradient field  $\langle 2x-4, 2y+2 \rangle$

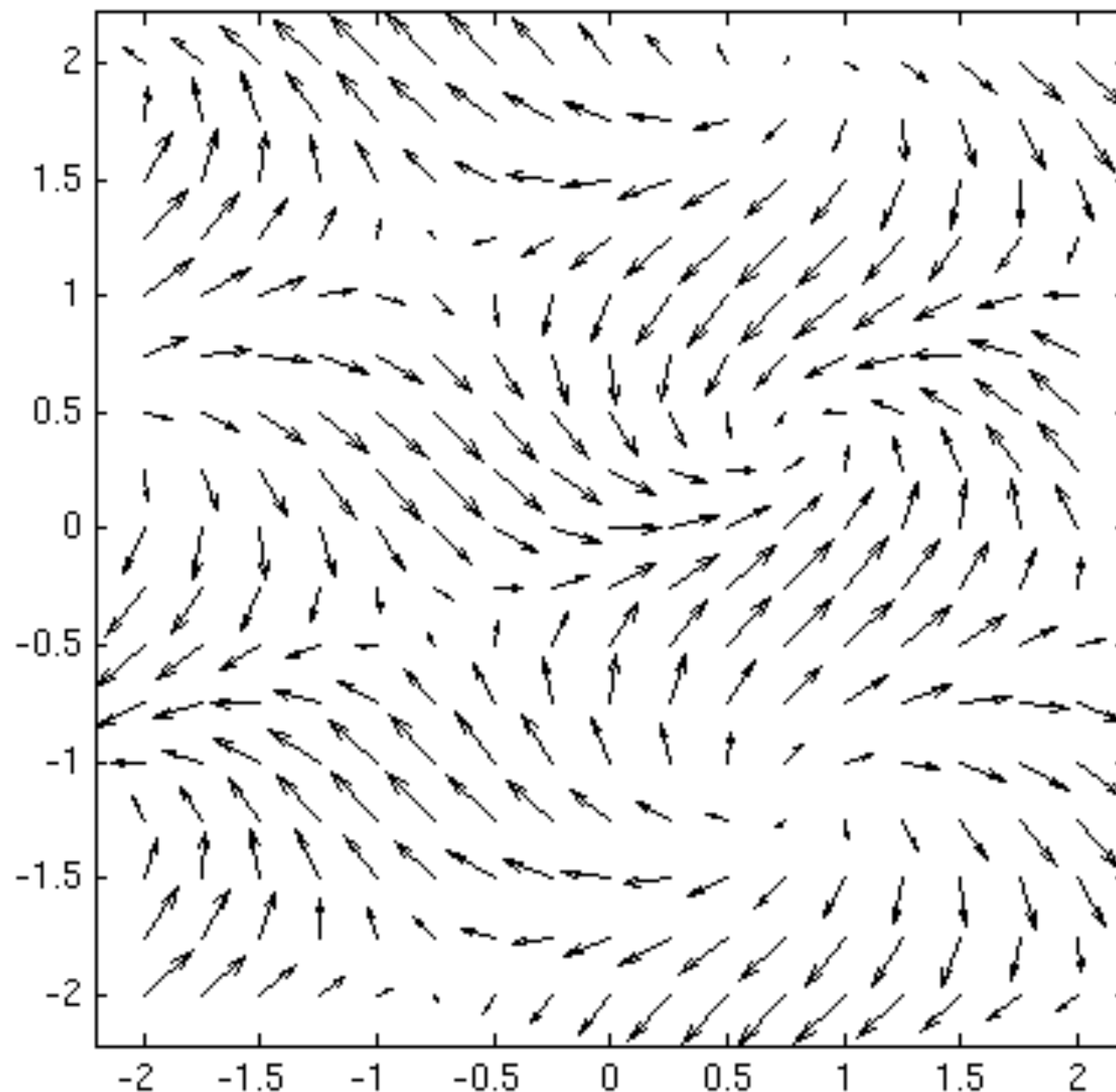


of the function  $f = x^2 - 4x + y^2 + 2y$ .





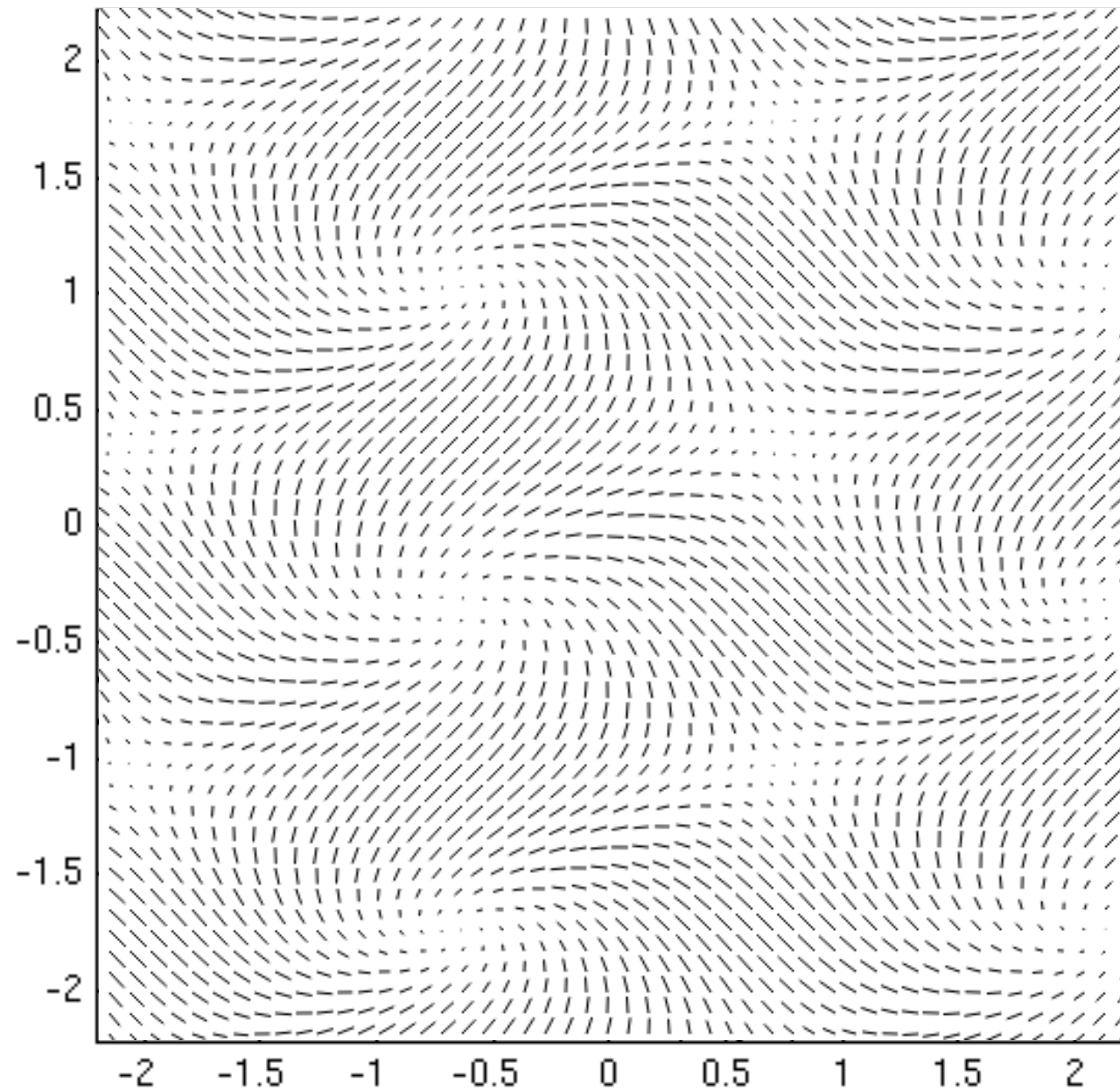
# Vector fields can be more complicated



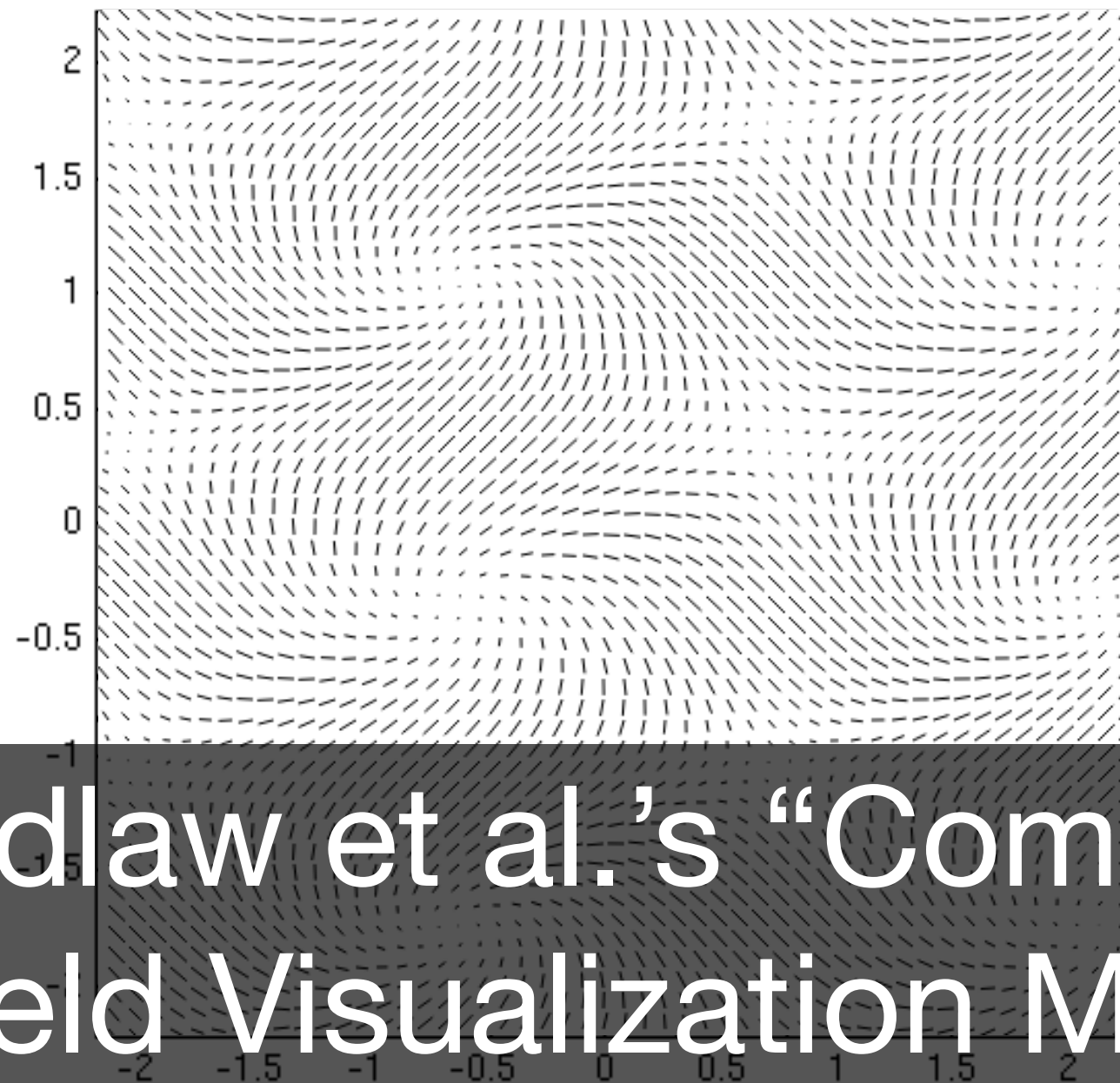
$$v(x, y) = (\cos(x + 2y), \sin(x - 2y))$$

# Glyph Based Techniques

# Hedgehog Plot: Not Very Good

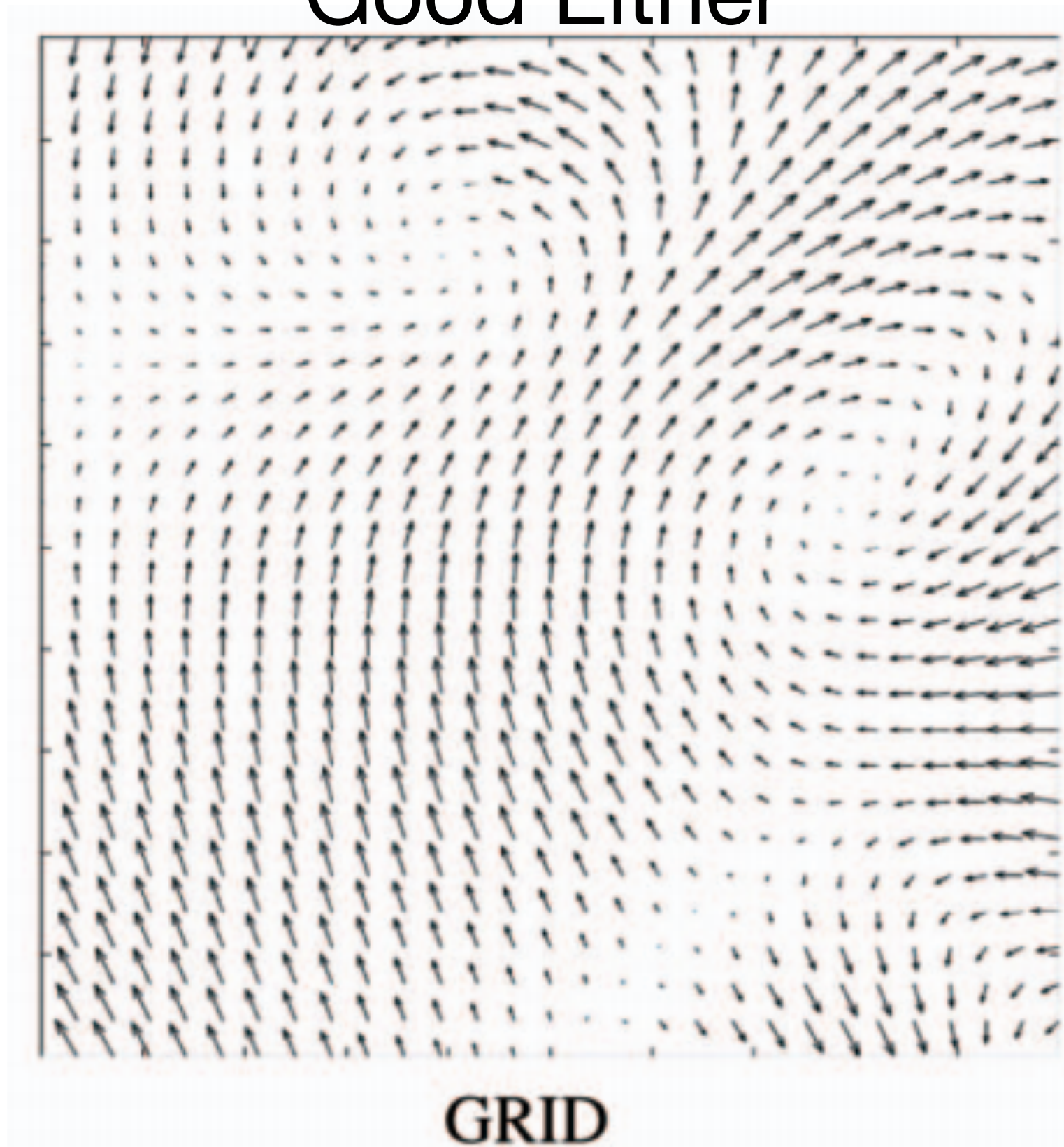


# Hedgehog Plot: Not Very Good



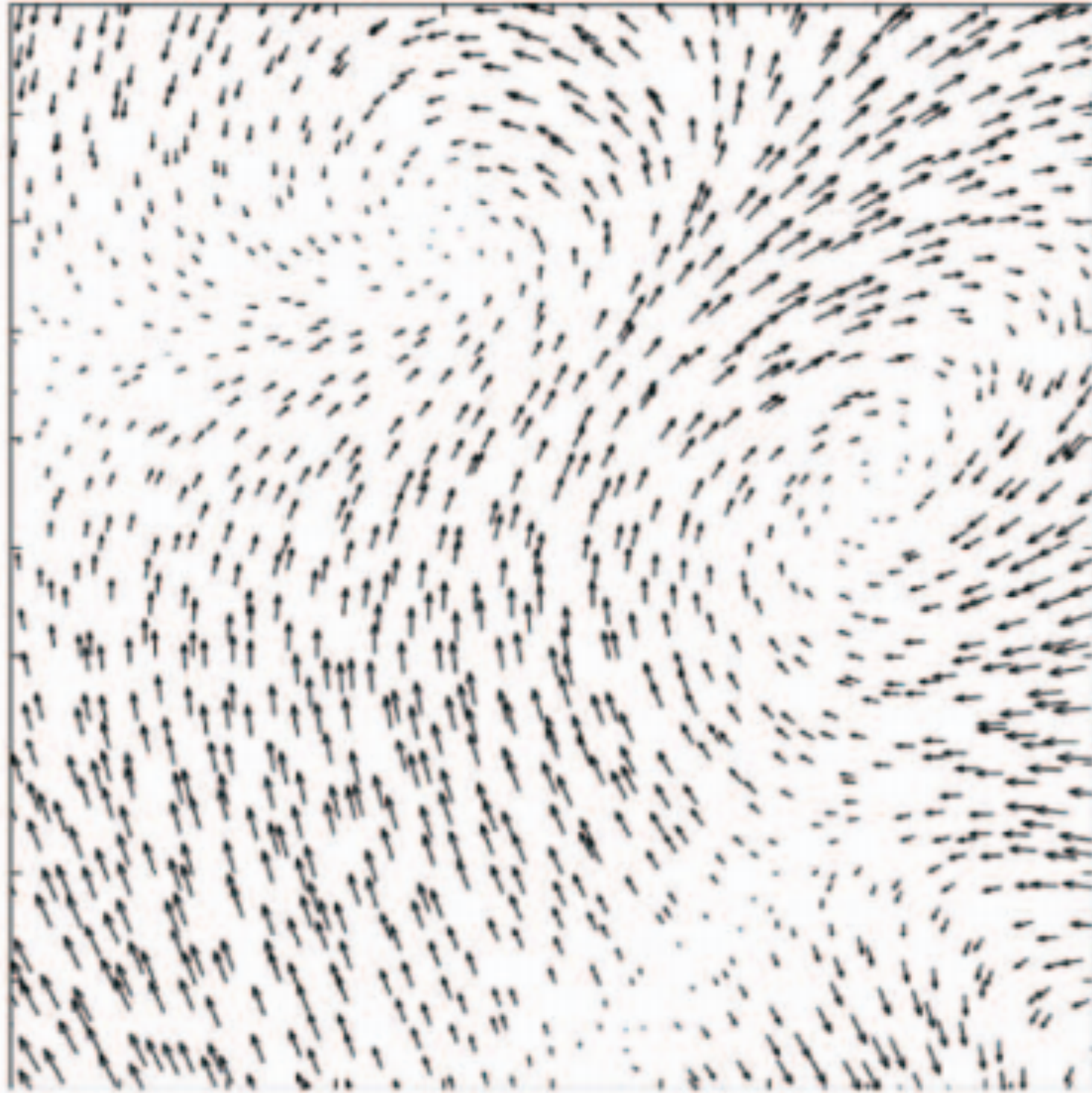
From Laidlaw et al.'s "Comparing 2D Vector Field Visualization Methods: A User Study", TVCG 2005

# Uniformly-placed arrows: Not Very Good Either





# Jittered Hedgehog Plot: Better



**JIT**



# Space-filling scaled glyphs





# Streamline-Guided Placement



OSTR



# Streamline-Guided Placement



OSTR

# Streamlines

# Streamlines





# Streamlines



**Curves everywhere tangent to the vector field**





# Curves everywhere tangent to the vector field



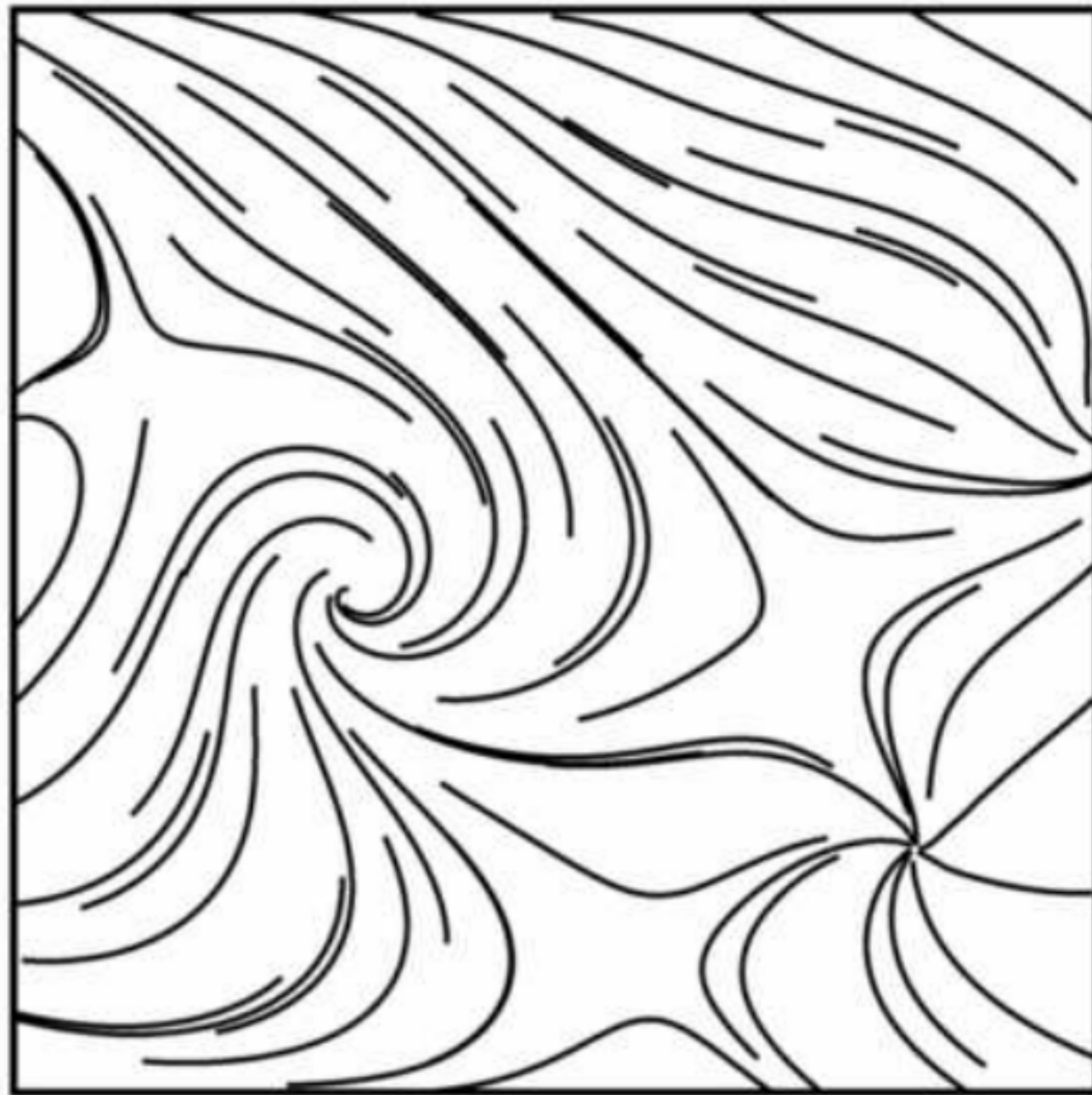
$$\begin{aligned}x'(t) &= v_x(x(t), y(t)) \\ y'(t) &= v_y(x(t), y(t))\end{aligned}$$



# Visualization via streamlines

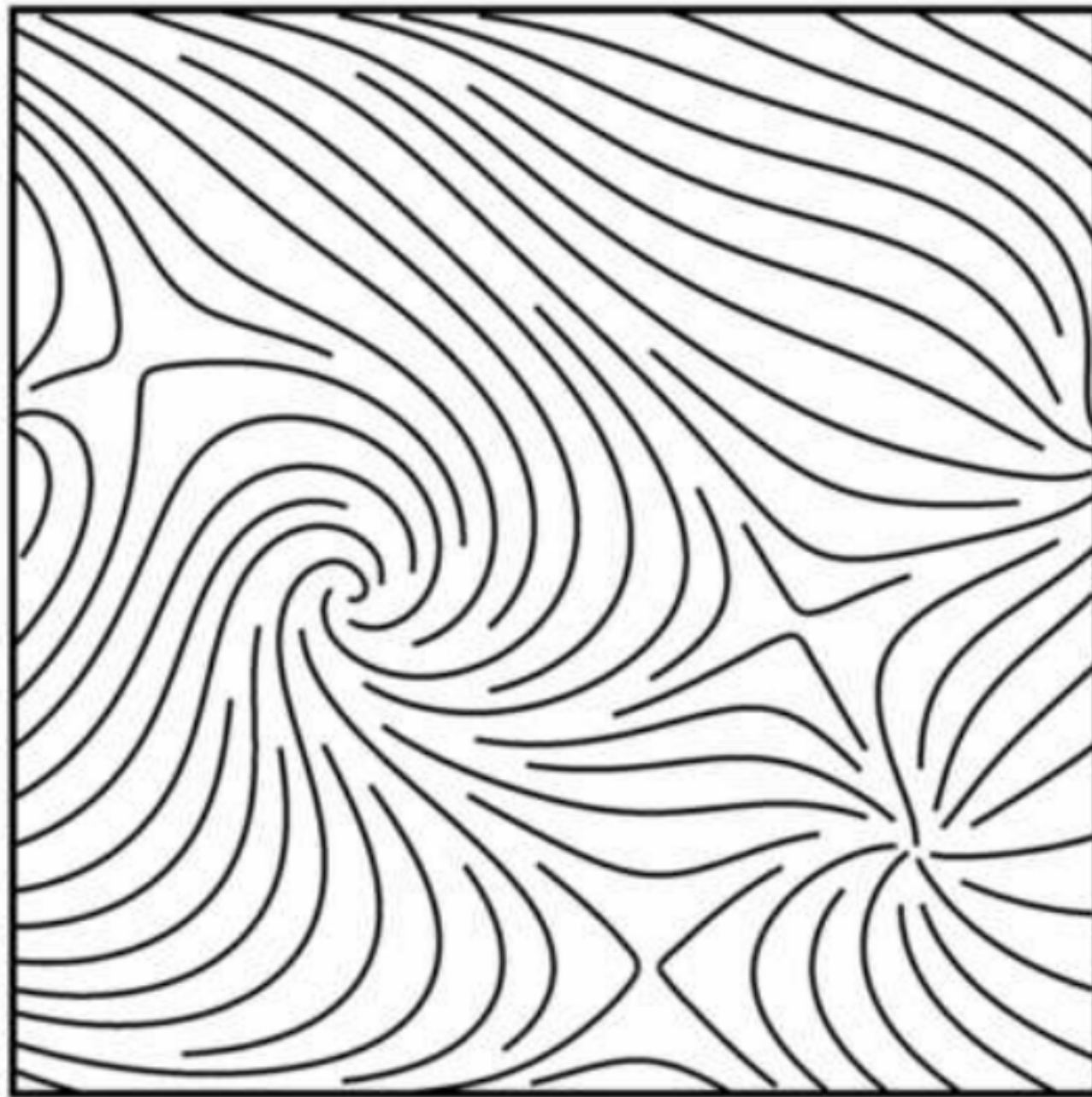
- Pick a set of seed points
- Integrate streamlines from those points
  - How do we compute this?
  - **[https://cscheid.net/writing/data\\_science/odes/index.html](https://cscheid.net/writing/data_science/odes/index.html)**
- **Which seed points?**

# Uniform placement



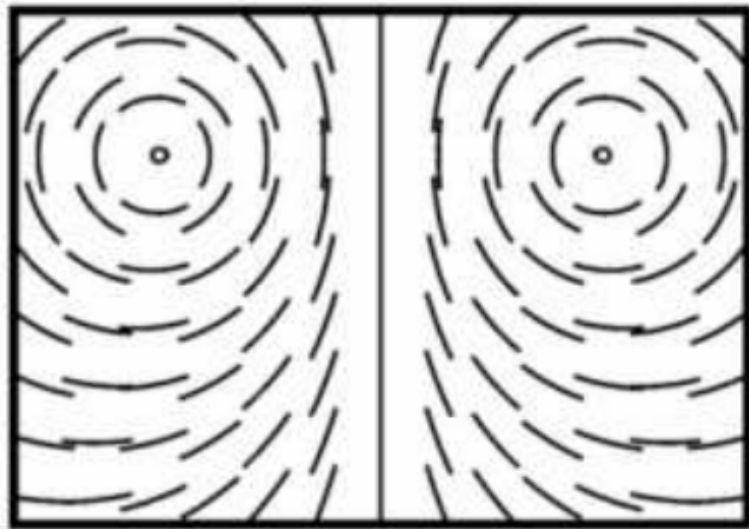
Turk and Banks, Image-Guided Streamline Placement  
SIGGRAPH 1996

# Density-optimized placement

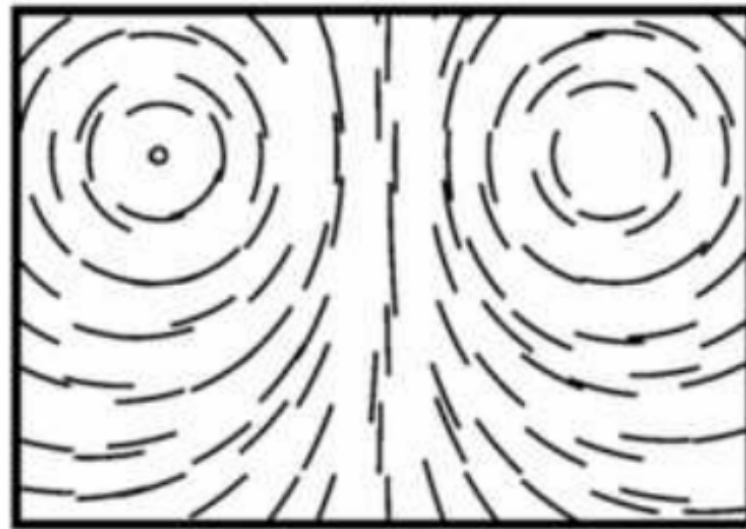


Turk and Banks, Image-Guided Streamline Placement  
SIGGRAPH 1996

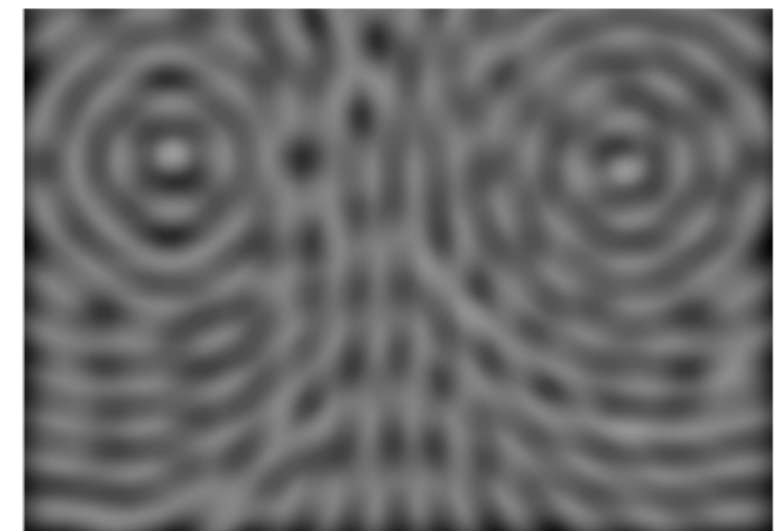
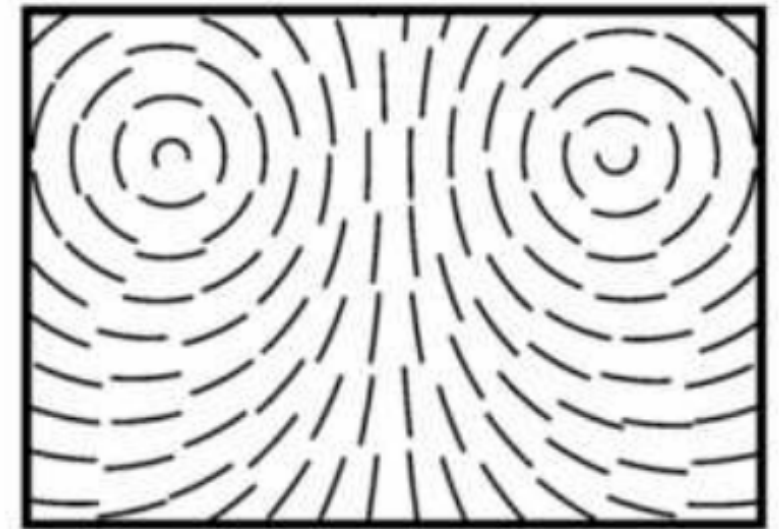
# Density-optimized placement



**Figure 2:** (a) Short streamlines with centers placed on a regular grid (top); (b) filtered version of same (bottom).



**Figure 3:** (a) Short streamlines with centers placed on a jittered grid (top); (b) filtered version showing bright and dark regions (bottom).



**Figure 4:** (a) Short streamlines placed by optimization (top); (b) filtered version showing fairly even gray value (bottom).

Turk and Banks, Image-Guided Streamline Placement  
SIGGRAPH 1996

# Image-Based Vector Field Visualization



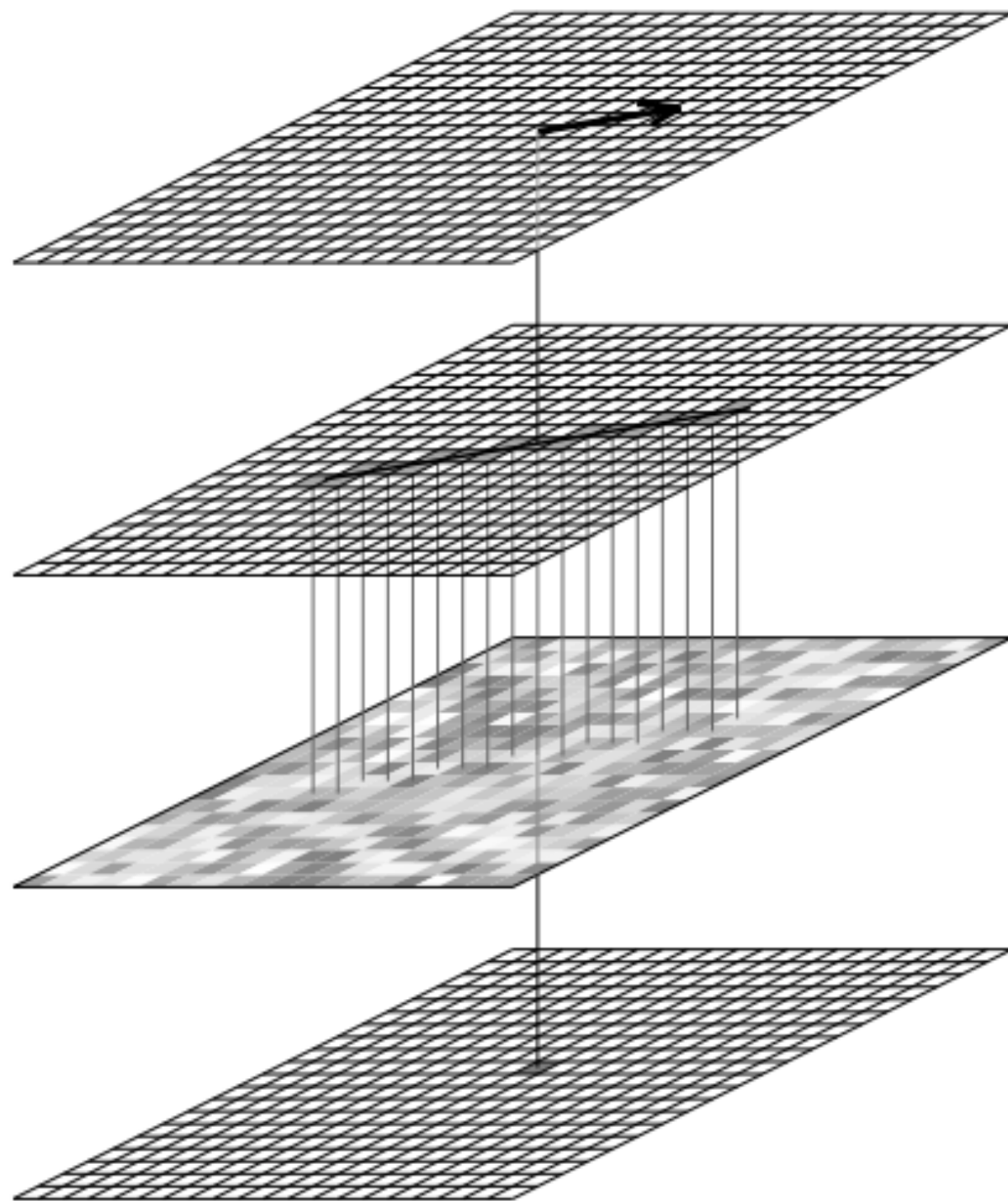
# Line Integral Convolution

<http://www3.nd.edu/~cwang11/2dflowvis.html>

Cabral and Leedom, Imaging Vector Fields using Line Integral Convolution. SIGGRAPH 1993



# Line Integral Convolution



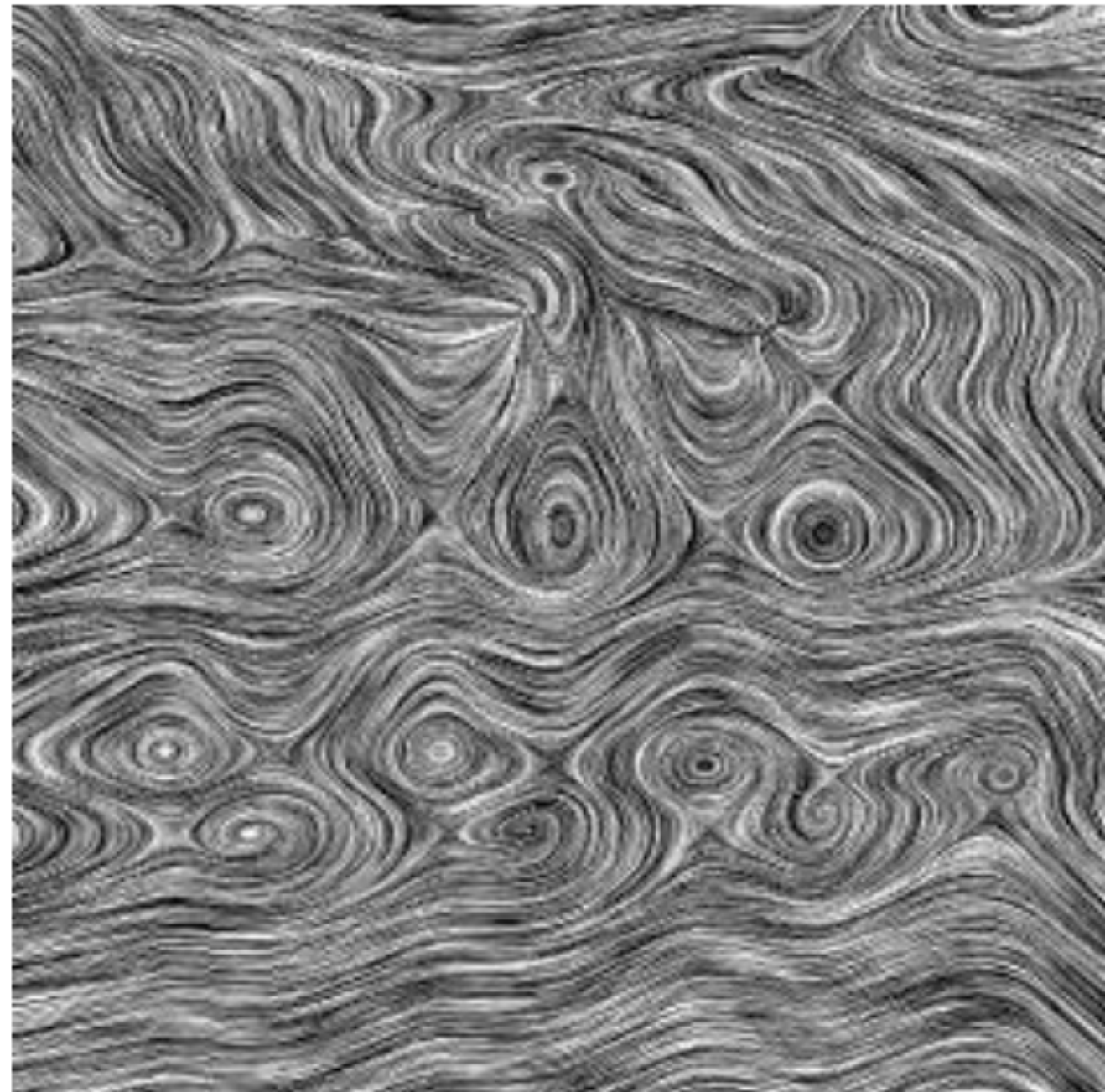
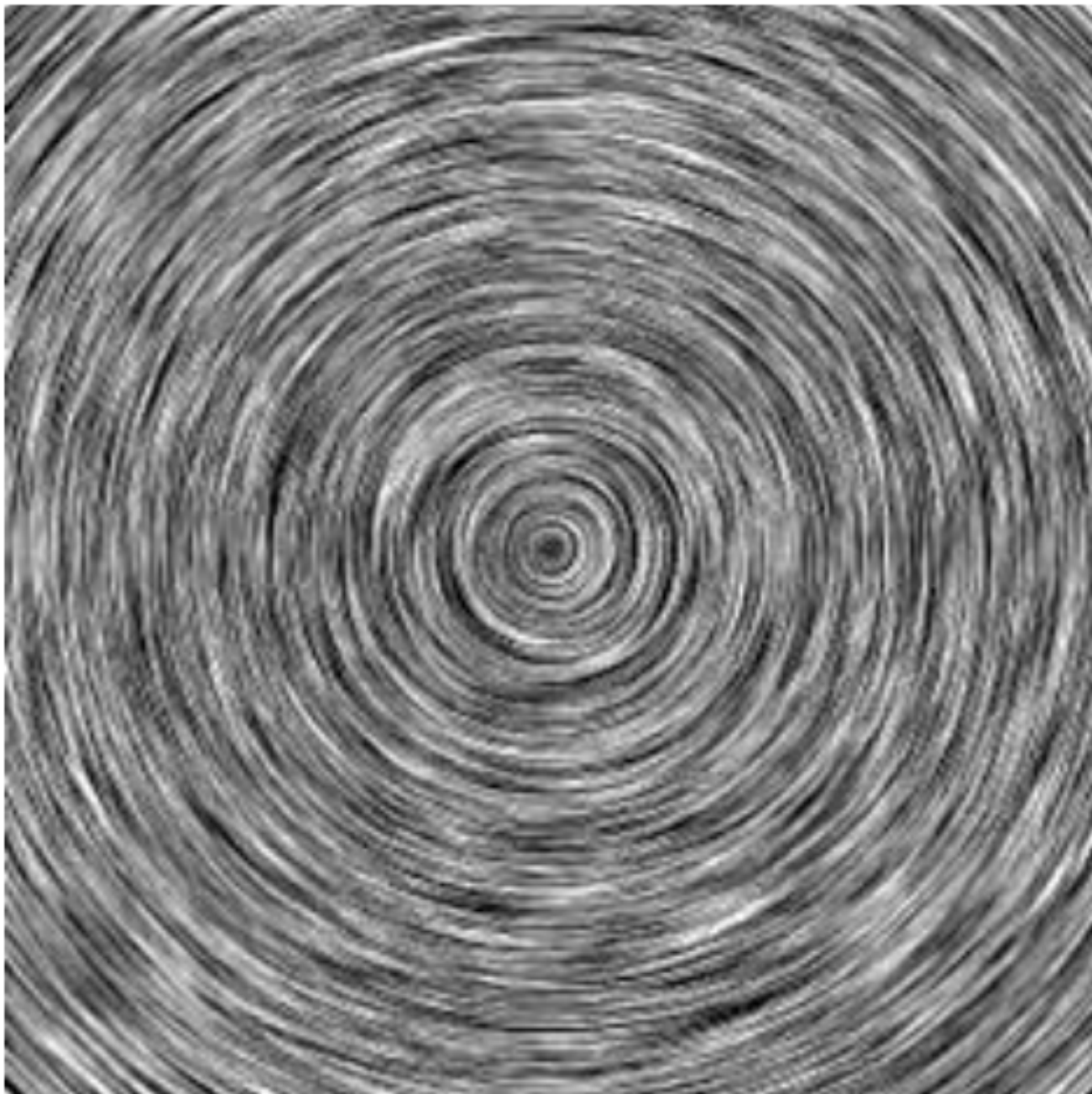
Given a  
**vector field**

compute  
**streamlines**

average  
**source of noise**  
along streamlines

**Result**

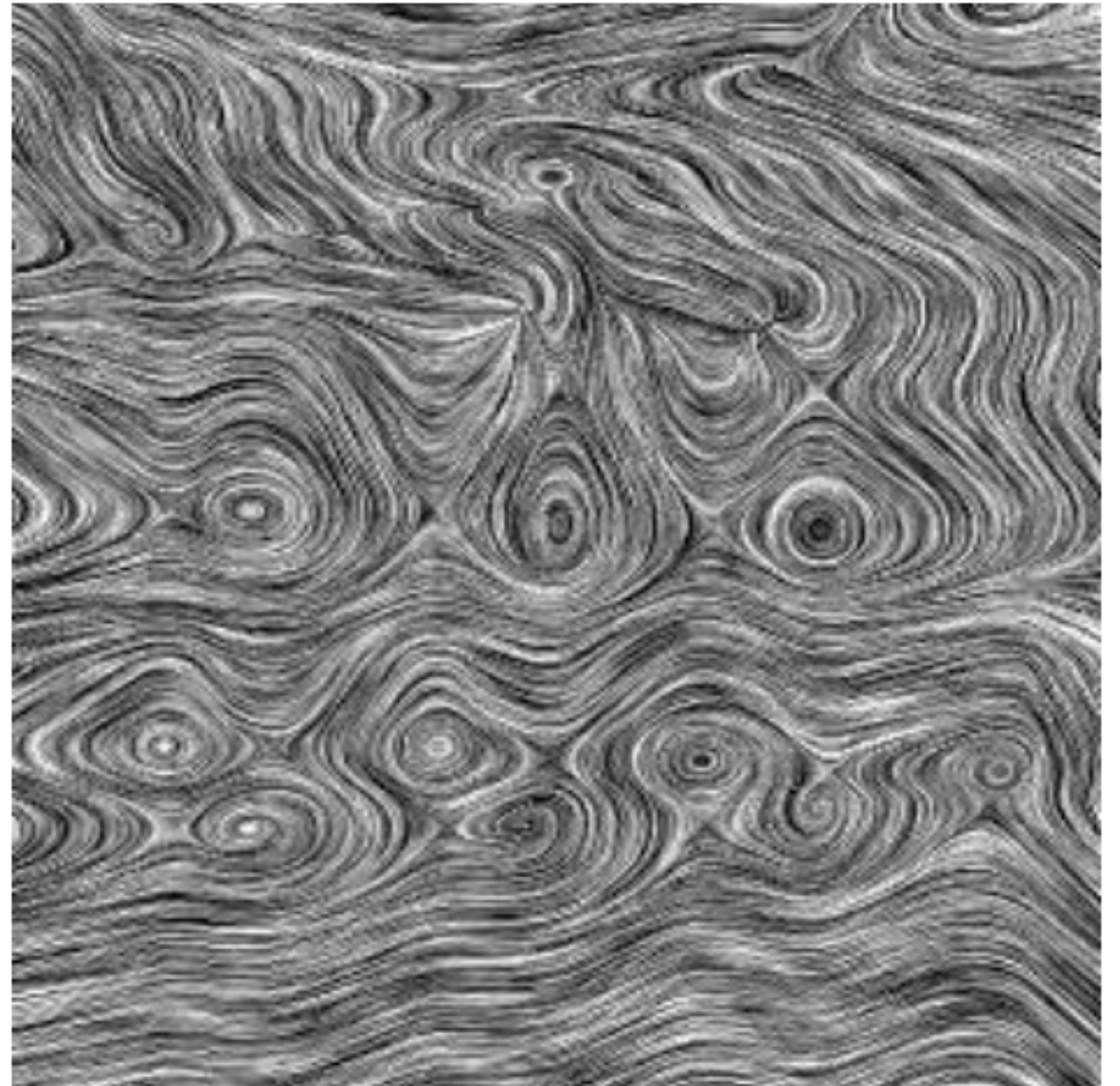
# Line Integral Convolution





# Advantages

- “Perfect” space usage
- Flow features are very apparent



# Downsides

- No perception of velocity!
- No perception of direction!

